COMP3610/6361 Principles of Programming Languages

Assignment 3

ver 1.0

Submission Guidelines

- Due time: Oct 12, 2023, 23:59pm (Canberra Time)
- Submit a pdf via Wattle.
- Scans of hand-written text are fine, as long as they are readable and neat.
- Please read and sign the declaration on the last page and attach a copy to your submission.
- **No late submission, deadline is strict**

Exercise 1 (Data Structure) (40 Marks)

Next to products, sums and records we want to extend our language by the data structure of binary trees. Nodes (and leaves) in the tree should carry values of type T.

The new syntax should be

$$
\begin{array}{llll}
E & ::= & \texttt{empty}[T] & \textit{// empty tree of type T} \\
 & & \texttt{fork}[T]\ E\ E\ E & \textit{//tree constructor of the form} < \textit{node} >< \textit{tree} >< \textit{tree} > \\
 & & \texttt{right}[T]\ E & \textit{//right subtree} \\
 & & \texttt{left}[T]\ E & \textit{//left subtree} \\
 & & \texttt{content}[T]\ E & \textit{//content of node} \\
 & & \texttt{isEmpty}[T]\ E & \textit{//test for emptyness} \\
 & & \texttt{isLeaf}[T]\ E & \textit{//test whether the tree consist of a single node only}
\end{array}
$$

Question 1 Define values and types necessary to define binary trees.

Question 2 Define meaningful semantics (small step) for trees, based on the given syntax. (The meaning of the expressions should be obvious.) You can use a variant of IMP that features error handling (you choose which kind).

Question 3 Provide typing rules for your semantics.

Question 4 Argue (or prove) that the progress and preservation theorems hold for your extension, when assuming our while language IMP as base, including booleans.

(Remember to justify your answers.)

Exercise 2 (Semantic Equivalence) (35 Marks)

Question 5 Prove cases “**if** _ **then** E_2 **else** E_3” and “**while** E_1 **do** _” of the Congruence theorem for semantic equivalence (Lecture 20/09).

Question 6 Prove that if $\Gamma_1 \vdash E_1 : \text{unit}$ and $\Gamma_2 \vdash E_2 : \text{unit}$ with Γ_1 disjoint from Γ_2 then, for $\Gamma = \Gamma_1 \cup \Gamma_2$,

$$E_1 \,;\, E_2 \simeq_{\Gamma}^{\text{unit}} E_2 \,;\, E_1 \;.$$

Question 7 Prove that the programs $l : \text{int ref} \vdash l := 0 : \text{unit}$ and $l : \text{int ref} \vdash l := 1 : \text{unit}$ are not contextually equivalent. Hint: find a context that will diverge for one of them, but not for the other.

Exercise 3 (Denotational Semantics) (25 Marks)

Question 8 Using denotational semantics (as defined in the lecture), prove that **skip** ; c and c ; **skip** are equivalent. That means, show

$$\mathscr{C}[\![\textbf{skip}\,;\, c]\!] = \mathscr{C}[\![c\,;\,\textbf{skip}]\!] \;.$$

Question 9 For the definition of the semantics of while, we used the function

$$\begin{aligned} F(f) = &\{(s,s) \mid (s,\texttt{false}) \in \mathscr{B}[\![b]\!]\} \cup \\ &\{(s,s') \mid (s,\texttt{true}) \in \mathscr{B}[\![b]\!] \wedge \\ &\qquad \exists s''.\,(s,s'') \in \mathscr{C}[\![c]\!] \wedge (s'',s') \in f\} \end{aligned}$$

(see lecture). By Kleene’s fixed point theorem we have that fix $(F) = \bigcup_{i\geq 0} F^i(\emptyset)$

- Prove that **while** `false` **do** c is equivalent to **skip**. Hint: prove (by induction) that $F^i(\emptyset) = \{(s,s)\}$.
- Calculate $\mathscr{C}[\![\textbf{while } \texttt{true} \textbf{ do skip}]\!]$ using the same technique.

Academic Integrity

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____________________ Date

____________________ Signature