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Finding multiple routing paths in wide-area WDM networks

Weifa Liang[a,*], Xiaojun Shen[b]

[a]Department of Computer Science, The Australian National University, Canberra, ACT 0200, Australia
[b]School of Interdisciplinary Computing and Engineering, University of Missouri-Kansas City, Kansas City, MO 64110, USA

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Abstract

In this paper a multiple routing path problem in wide area Wavelength Division Multiplexing (WDM) networks is considered, which is to find K edge-disjoint lightpaths/semilightpaths from a source to a destination, if they exist, such that they meet some specified optimization objective. Two versions of the problem are studied. One is to minimize the total cost of the K paths, and the other is to minimize the cost of the maximum cost one among the K paths. An efficient algorithm for the first version is proposed, which takes $O(kK(kn+m+n\log(kn)))$ time and delivers an exact solution, where n, m, and k are the number of nodes, links and wavelengths in the network, respectively. The second version of the problem is shown to be NP-hard, instead an approximation algorithm is devised which delivers a solution within K times of the optimum, where $K \geq 2$.

Keywords: WDM opticla networks robust routing; Combinatorial optimization

1. Introduction

It becomes increasing evident that wavelength-division multiplexing (WDM) providing multigigabit rates per wavelength will soon become the core technology for the next-generation Internet [8]. The emerging WDM optical network offers the possibility of interconnecting hundreds of thousands of users, covering local to wide area. The key to the high speed in the network is to maintain the signal in optical form rather than traditionally electronic form. The high bandwidth of fiber-optic links is utilized through *Wavelength-Division Multiplexing* (WDM) technology, which supports propagating multiple laser beams through a single fiber-optic link, provided that each laser beam uses a distinct optical wavelength. The major applications of this type of network are video conferencing, scientific visualization, real-time medical imaging, supercomputing, and distributed computing [2,15,18]. A comprehensive overview of its physical theory can be found in [9,14].

Routing in a WDM network is a fundamental problem. The data transfer in the network is through first establishing a lightpath and then proceeding the transfer. Lightpaths thus provide a powerful approach to utilize the vast available bandwidth in optical networks [1,5,10], while a lightpath is implemented by assigning a unique wavelength to all the links in the path. Data transmitted through a lightpath does not need wavelength conversion or electronic processing at intermediate nodes. Although transmitting all traffic between every pair of nodes over lightpaths is desirable, it is not generally feasible to establish such lightpaths and accommodate the traffic by the lightpaths due to physical constraints imposed by the network such as the limited number of wavelengths, limited tunability of optical transceivers at each node. To cope with these limits, Chlamtac et al. [4] introduced the *semilightpath* concept, which is a transmission path by chaining several lightpaths together. Therefore, for a semilightpath, the wavelength conversions at some intermediate nodes are required. The cost of a semilightpath/lightpath is the sum of the costs of its links and nodes, where the link cost is associated with traversing the link using some wavelength, and the node cost is associated with wavelength conversion when it has to switch to a different wavelength at the node.

* Corresponding author. Tel.: +61 26 125 3019; fax: +61 26 125 0010.
E-mail addresses: wliang@cs.anu.edu.au (W. Liang), xshen@cstp.umkc.edu (X. Shen).

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Thus, a minimum cost semilightpath is called the *optimal semilightpath*, for which Chlamtac et al. [4] presented an $O(k^2n+kn^2)$ time algorithm, Liang et al. [12,13] later gave an improved algorithm, which requires $O(k^2n+km+kn\log(kn))$ time, where n is the number of nodes, m is the number of physical optic links, and k is the number of wavelengths in the network.

In this paper we consider a multiple routing path problem as follows. Given a WDM network and a source s and a destination t, find K edge-disjoint semilightpaths from s to t such that the K paths meet some specified optimization objective, where $K\geq 2$. Here we deal with two versions of the problem. The first version is to minimize the total cost of the K semilightpaths and the second version is to minimize the cost of the maximum cost one among the K semilightpaths. This is a fundamental problem in communications networks and has wide application backgrounds. For example, the multiple routing paths from a source to a destination is fault-tolerant and the data from the source to the destination can still be delivered even if there are r semilightpaths failures ($r<K$). Also, in some real-time critical applications, it is necessary to establish multiple semilightpaths from a source to a destination for data transmission to guarantee the quality of service (QoS).

In this paper our major contributions are as follows. The two versions of the multiple routing path problem have been formulated. An efficient algorithm for the first version is presented, which takes $O(kK(kn+m+n\log(kn)))$ time and delivers an exact solution. The second version of the problem is shown to be NP-hard, and an approximation algorithm is devised. The solution delivered by the approximation algorithm is K times of the optimum, where $K\geq 2$.

The rest of this paper is organized as follows. In Section 2 the network model is provided, followed by the introduction of the measure parameters used in the network. The multiple routing path problem is also defined precisely. In Section 3 an auxiliary weighted directed graph is defined. In Section 4 the first version of the problem is considered and an exact algorithm is presented. In addition, the second version of the problem is shown to be NP-hard and an approximation solution is proposed. In Section 5 the distributed implementation issues of the proposed algorithms are discussed. The conclusion is given in Section 6.

2. Preliminaries

The optical network is modeled by a *directed graph* $G=(V,E,\Lambda)$, where V is the set of nodes (vertices), E is the set of directed links (edges), and Λ is a set of wavelengths in G, $n=|V|$, $m=|E|$, and $|\Lambda|=k\Lambda=\{\lambda_1,\lambda_2,\ldots,\lambda_k\}$. Associated with each node $v\in V$, there is a switch converter, which can convert an incoming wavelength to another outgoing wavelength if necessary. The switching operation at a node uses a wavelength conversion table and this table is given in advance. Associated with each link $e\in E$, there is a set $\Lambda(e)$ $(\subseteq\Lambda)$ of wavelengths available on it initially.

Following the cost definition of semilightpath by Chlamtac et al. [4], the *cost structure* of using network resources in G is defined as follows. For each link e and wavelength $\lambda_i\in\Lambda(e)$ a nonnegative weight $w(e,\lambda_i)$ is associated, representing the 'cost' of using wavelength λ_i on link e. The 'cost' of wavelength conversion is modeled via cost factors of the form $c_v(\lambda_p,\lambda_q)$, which is the cost of wavelength conversion at node v from wavelength λ_p to wavelength λ_q. If $\lambda_p=\lambda_q$, then $c_v(\lambda_p,\lambda_q)=0$.

A *semilightpath* $\mathcal{P}$ in G is a sequence $e_1,e_2,\ldots,e_l$ of directed links such that the tail of e_{i+1} coincides with the head of e_1, $i=1,\ldots,l$. Furthermore, a specific wavelength $\lambda_{j_i}\in\Lambda(e_i)$ is associated with each e_i that is the wavelength used on link e_i in the path. Denote by *head*(e) and *tail*(e) the head and tail of a directed link e, which are the two endpoints of link e. The *cost* C ($\mathcal{P}$) of the semilightpath $\mathcal{P}$ is $C(\mathcal{P})=\sum_{i=1}^{l}w(e_i,\lambda_{j_i})+\sum_{i=1}^{l-1}c_{\mathrm{head}(e_i)}(\lambda_{j_i},\lambda_{j_{i+1}})$. The optimal semilightpath $\mathcal{P}$ from s to t is such a path that C ($\mathcal{P}$) is minimized. To solve this problem, not only do we need to find an optimal semilightpath, but also do we need to assign every link e in the path a specific wavelength $\lambda(e)\in\Lambda(e)$ and to set the wavelength conversion switch at some intermediate nodes if needed.

Given two semilightpaths PH_1 and PH_2 in G from s to t, they are 'edge-disjoint' if they share the same node v, then the links in PH_1 and PH_2 either entering into or leaving from v are assigned with different wavelengths.

Let $PH_1^{(K)},PH_2^{(K)},\ldots,PH_K^{(K)}$ be the K edge-disjoint semilightpaths in G from s to t. The K semilightpaths are edge-disjoint if any two semilightpaths $PH_i^{(K)}$ and $PH_j^{(K)}$ are edge-disjoint for any i and j with $i\neq j$, $1\leq i,j\leq K$. We are now ready to define the two versions of the problem as follows.

Version 1. Given a WDM network $G(V,E,\Lambda)$ and a pair of nodes s and t, assume there is a wavelength conversion table at each node $v\in V$. The problem is to find K edge-disjoint semilightpaths in G from s to t such that the cost sum of the K paths is minimized, if they exist. In other words, let $PH_1^{(K)},PH_2^{(K)},\ldots,PH_K^{(K)}$ be the K edge-disjoint semilightpaths from s to t and $l_i^{(K)}$ the cost of $P_i^{(K)}$, $1\leq i\leq K$. The objective is to find the K semilightpaths subject to minimizing $\sum_{i=1}^{K}l_i$.

Version 2. Given a WDM network $G(V,E,\Lambda)$ and a pair of nodes s and t, assume there is a wavelength conversion table at each node $v\in V$. The problem is to find K edge-disjoint semilightpaths in G from s to t such that the cost of a maximum cost one among the K semilightpaths is minimized. Let $Q_1^{(K)},Q_2^{(K)},\ldots,Q_K^{(K)}$ be the K semilightpaths from s to t and $t_1^{(K)},t_2^{(K)},\ldots,t_K^{(K)}$ the costs of the K semilightpaths. The objective is to find the K semilightpaths subject to minimizing $\max\{t_i^{(K)}:1\leq i\leq K\}$.

3. Constructing auxiliary graph $G_{s,t}^{\text{edge}}$

In [12,13], we reduced the optimal semilightpath problem to a single-source shortest paths problem. Thus, a solution of this latter problem corresponds to a solution of the original problem. Following the same spirit, we here reduce the multiple routing path problem to a well solved optimization problem in an auxiliary weighted, directed graph $G_{s,t}^{\text{edge}}$, and the solution of the optimization problem in $G_{s,t}^{\text{edge}}$ gives an exact and approximate solutions for the two versions of the multiple routing path problem. The auxiliary graph $G_{s,t}^{\text{edge}}$ is constructed as follows.

Given a WDM network $G(V,E,\Lambda)$, there is a set $\Lambda(e)$ ($\subseteq \Lambda$) of wavelengths for each link $e \in E$. For each node $v \in V$, let $\Lambda_{\text{in}}(G,v)$ and $\Lambda_{\text{out}}(G,v)$ be the sets of incoming and outgoing wavelengths at it, then, $\Lambda_{\text{in}}(G,v) = \cup_{e \in E \& \text{head}(e)=v} \Lambda(e)$ and $\Lambda_{\text{out}}(G,v) = \cup_{e \in E \& \text{tail}(e')=v} \Lambda(e')$. A *directed* weighted graph $G_v = (X_v^1 \cup X_v^2 \cup Y_v^1 \cup Y_v^2, E_v, \omega_1)$ for each node $v \in V$ is constructed, where $X_v^1 \cup X_v^2 \cup Y_v^1 \cup Y_v^2$ is the set of nodes and E_v is the set of links in G_v. $E_v \subseteq (X_v^1 \times X_v^2) \cup (X_v^2 \times Y_v^1) \cup (Y_v^1 \times Y_v^2)$, and $\omega_1 : E_v \to R$ is a weight function of links in G_v. For each distinct $\lambda \in \Lambda_{\text{in}}(G,v)$, a corresponding node $x^{(1)}$ is in X_v^1 and another corresponding node $x^{(2)}$ is in X_v^2. There is a directed link $\langle x^{(1)}, x^{(2)} \rangle \in E_v$ with weight 0. For each distinct $\lambda' \in \Lambda_{\text{out}}(G,v)$, a corresponding node $y^{(1)}$ is in Y_v^1 and another corresponding node $y^{(2)}$ is in Y_v^2. There is a directed link $\langle y^{(1)}, y^{(2)} \rangle \in E_v$ with weight 0. In addition, there is a directed link $e = \langle x^{(2)}, y^{(1)} \rangle \in E_v$ from $x^{(2)} \in X_v^2$ to $y^{(1)} \in Y_v^1$ and the weight assigned to it is $\omega_1(e) = C_v(\lambda,\lambda) = 0$ if $\lambda = \lambda'$; the weight is $\omega_1(e) = C_v(\lambda,\lambda')$, otherwise.

The directed, weighted auxiliary graph $G_{s,t}^{\text{edge}} = (V', E', \omega_2)$ is then constructed as follows. $\omega_2 : E' \to R$ is the weight function of links in $G_{s,t}^{\text{edge}}$, using the information supplied by G and G_v for all $v \in V$. $V' = \cup (X_v^1 \cup X_v^2 \cup Y_v^1 \cup Y_v^2) \cup \{s', t''\}$, where s and t'' are the two special nodes, which represent the source s and destination t, respectively. $E' = \cup_{v \in V} E_v \cup \{\langle s', v \rangle : v \in Y_s^1\} \cup \{\langle u, t'' \rangle : u \in X_t^1\} \cup E_\Lambda$, and E_Λ is defined as follows. Let $\lambda \in \Lambda(e)$ and $e = \langle u,v \rangle \in E$. Then, there are two nodes $u' \in Y_u^2$ and $v' \in X_v^1$ in G_v and G_u that correspond to λ. Following the construction of G_u and G_v, there is a link $\langle u', v' \rangle \in E_\Lambda$ and its weight is $\omega_2(\langle u', v' \rangle) = \omega(\langle u,v \rangle, \lambda)$. The weights associated with links $\langle s', v \rangle$ and $\langle u, t'' \rangle$ are 0 s, for every $v \in Y_s^1$ and every $u \in X_t^1$. The weight associated with every link $\langle u, v \rangle \in \cup_{v \in V} E_v$ is $\omega_2(u,v) = \omega_1(u,v)$. $G_{s,t}^{\text{edge}}$ contains no more than $4kn+2$ nodes and $k^2n + km + 4k$ links, because $|V'| = \sum_{v \in V}(|X_v^1| + |X_v^2| + |Y_v^1| + |Y_v^2|) + 2 = 2\sum_{v \in V}(|\ _{\Lambda \text{in}}(G,v)| + |\Lambda_{\text{out}}(G,v)|) + 2 \le 4kn + 2$, and $|E'| = \sum_{v \in V} |E_v| + |Y_s^1| + |X_t^1| + |E_\Lambda| \le (k^2 n + 2k) + 2k + \sum_{e \in E} |\Lambda(e)| \le k^2 n + km + 4k$.

Let P be a directed path in $G_{s,t}^{\text{edge}}$ from s' to t'' and $e'_1, e'_2, e'_3, e'_4, e'_5, e'_6, e'_7, e'_8, e'_9, e'_{10}, \ldots, e'_{4i+3}, e'_{4i+4}, e'_{4i+5}, e'_{4i+6}, \ldots, e'_l$ the sequence of links in P, $0 \le i \le (l/4($. For any given link e'_j in P, link e'_{j-1} is called *the immediate predecessor* of e'_j and e'_{j+1} is called *the immediate successor* of e'_j if they exist. In the following it shows that every directed path P in $G_{s,t}^{\text{edge}}$ from s' to t'' corresponds to a semilightpath PH in G from s to t.

Following the construction of $G_{s,t}^{\text{edge}}$, e'_1 and e'_2 are derived from the source node of PH, and e'_l is derived from the destination node of PH. e'_3 is a link induced from the two nodes in G and e'_4, e'_5, and e'_6 are the links derived from a node in G, and e'_5 corresponds to a wavelength conversion at the node. Similarly, e'_7 is a link induced from the two nodes in G, e'_8, e'_9, and e'_{10} are the links derived from a node in G, and e'_9 corresponds to a wavelength conversion at the node. In general, e'_{4i+3} is a link induced from the two nodes in G and e'_{4i+4}, e'_{4i+5}, and e'_{4i+6} are the links generated from a node in G, and e'_{4i+5} corresponds to a wavelength conversion at the node, $0 \le i \le (l/4($. Therefore, for every i, $0 \le i \le (l/4($, e'_{4i+3} corresponds to a directed link e_j in G with weight $\omega_2(e'_{4i+3}) = w(e_j, \lambda)$, and e_j is a link in PH with wavelength λ. e'_{4i+4}, e'_{4i+5} and e'_{4i+6} correspond to a wavelength conversion at node v from wavelength λ to wavelength λ', if the weight of e'_{4i+5} is $\omega_2(e'_{4i+5}) = c_v(\lambda, \lambda')$. Therefore, a semilightpath in G from s to t consists of $(l/4($ physical optical links. Fig. 1 is a WDM network, where $\Lambda = \{\lambda_1, \lambda_2, \lambda_3, \lambda_4\}$ is the wavelength set in G. The wavelength set on each link in G is as follows. $\Lambda(\langle 1,2 \rangle) = \{\lambda_1, \lambda_3\}$, $\Lambda(\langle 1,4 \rangle) = \{\lambda_1, \lambda_2, \lambda_4\}$, $\Lambda(\langle 2,3 \rangle) = \{\lambda_1, \lambda_4\}$, $\Lambda(\langle 3,1 \rangle) = \{\lambda_2, \lambda_3\}$, $\Lambda(\langle 4,5 \rangle) = \{\lambda_3\}$, $\Lambda(\langle 5,3 \rangle) = \{\lambda_2, \lambda_4\}$. The wavelength conversion table at each node is as follows. At node 1, we have $c_1(\lambda_2,\lambda_1)$, $c_1(\lambda_2,\lambda_2)$, $c_1(\lambda_2,\lambda_4)$, $c_1(\lambda_3,\lambda_3)$, and $c_1(\lambda_3,\lambda_4)$. At node 2, we have $c_2(\lambda_1,\lambda_1)$, $c_2(\lambda_1,\lambda_4)$, and $c_2(\lambda_3,\lambda_1)$. At node 3, we have $c_3(\lambda_1,\lambda_2)$, $c_3(\lambda_1,\lambda_3)$, $c_3(\lambda_2,\lambda_2)$, and $c_3(\lambda_4,\lambda_3)$. At node 4, we have $c_4(\lambda_1,\lambda_3)$ and $c_4(\lambda_4,\lambda_3)$. At node 5, we have $c_5(\lambda_3,\lambda_2)$ and $c_5(\lambda_3,\lambda_4)$. Fig. 2 illustrated the construction of $G_3 = (X_3^1 \cup X_3^2 \cup Y_3^1 \cup Y_3^2, E_3, \omega_1)$ for node 3 in G, where a node labeled by '(v,λ_j)' means that the node is derived from node v in G and wavelength λ_j. From Fig. 2 we can see that there is not any link from a node in X_3^2 labeled by $(3,\lambda_2)$ to a node in Y_3^1 labeled by $(3,\lambda_3)$, which means that the wavelength conversion from λ_2 to λ_3 at node 3 is not allowed. Fig. 3 provides a subgraph G' of $G_{s,t}^{\text{edge}}$ induced by the nodes in G_1 and G_3 and a link in G between nodes 1 and

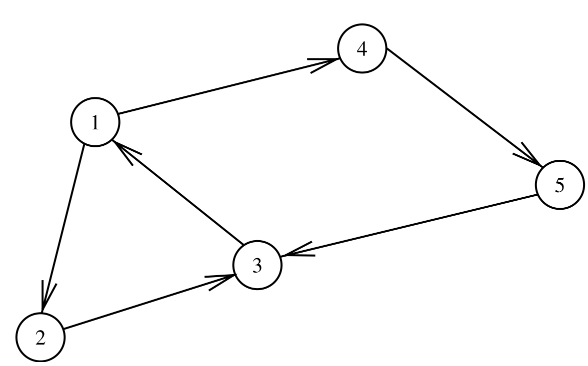

Fig. 1. The WDM network $G(V,E)$.

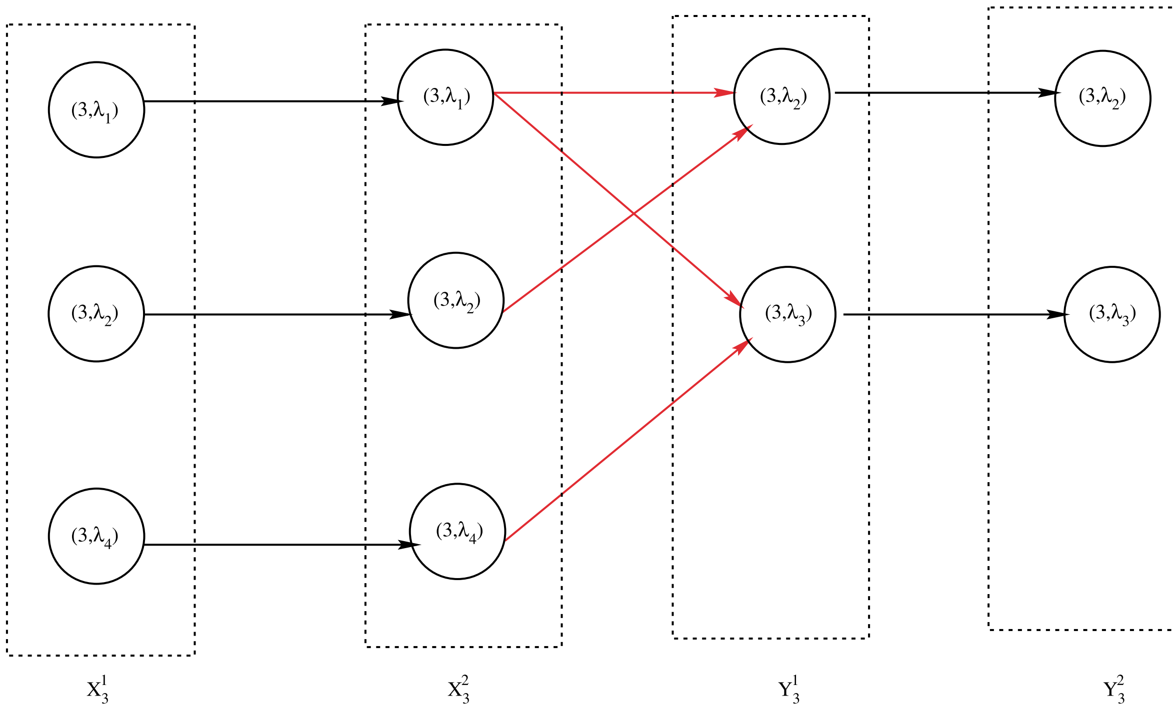

Fig. 2. The auxiliary graph $G_3=(X_3^1\cup X_3^2\cup Y_3^1\cup Y_3^2, E_3, \omega_1)$ at node 3.

3. Note that each of the two links $\langle u,v\rangle$ and $\langle p,q\rangle$ in Fig. 3 is a link from a node in G_3 to a node in G_1, which is derived from link $\langle 3,1\rangle$ in G with $\Lambda(\langle 3,1\rangle)=\{\lambda_2,\lambda_3\}$.

Given a source $s(=v_2)$ and a destination $t(=v_4)$ in G, the auxiliary graph $G_{s,t}^{\text{edge}}$ of G is shown in Fig. 4, where a node labelled by λ_i within a dotted rectangle v_j should be interpreted as that node is derived from wavelength λ_i and node $v_j\in V$ in G.

4. Finding *K* edge-disjoint semilightpaths

4.1. K edge-disjoint semilightpaths with minimizing the total cost

Given a directed graph H and a pair of nodes s and t, if there are two directed paths in H from s to t such that they do not share edge (link), then they are 'edge-disjoint'. In order to

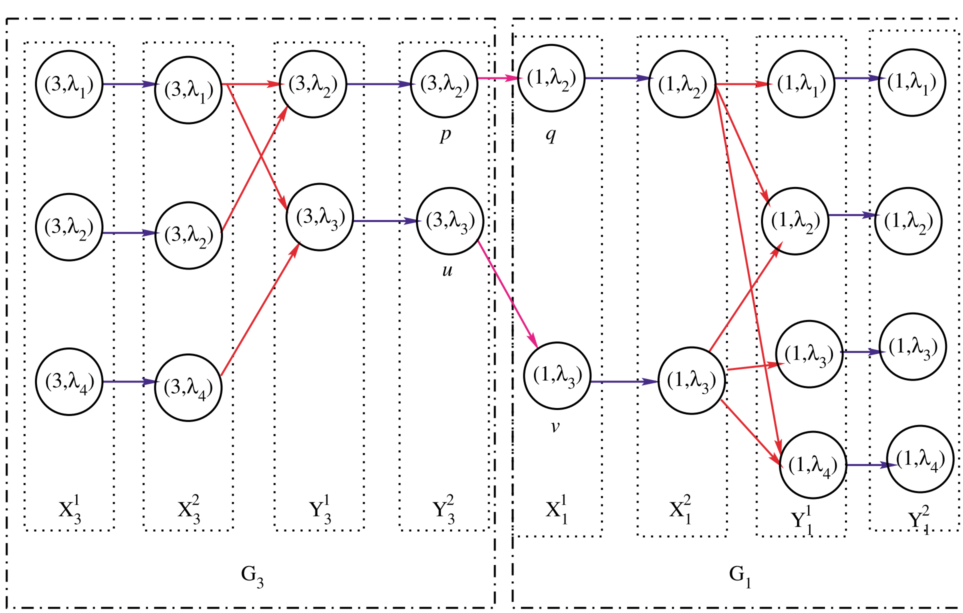

Fig. 3. A subgraph of $G_{s,t}^{\text{edge}}$ induced by the nodes in G_1 and G_3.

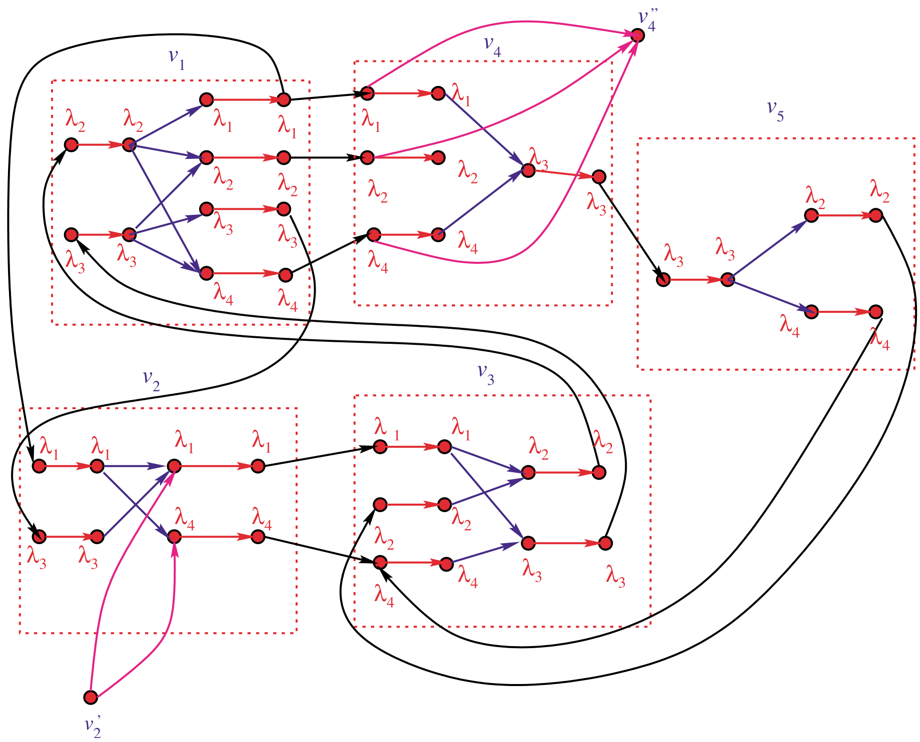

Fig. 4. The graph $G_{s,t}^{\text{edge}}$ with $s=v_2$ and $t=v_4$.

find K edge-disjoint semilightpaths in G from s to t with the minimization of the total cost of the K paths, the problem can be reduced to find K edge-disjoint paths in $G_{s,t}^{\text{edge}}$ from s' to t'' such that the total cost of the K paths is minimized. Therefore, if there are such K edge-disjoint paths in $G_{s,t}^{\text{edge}}$ from s' to t'', then there are K corresponding edge-disjoint semilightpaths in G from s to t and the total cost of the K semilightpaths is minimum. We, therefore, focus on finding K edge-disjoint paths in $G_{s,t}^{\text{edge}}$ from s' to t'' with an objective to minimizing the total cost of the K paths as follows.

For a given pair of nodes s' and t'', finding K edge-disjoint paths $P_1,P_2,\ldots,P_K$ in $G_{s,t}^{\text{edge}}$ from s' to t'' such that $\sum_{i=1}^{K} l_i$ is minimized can be done by an efficient algorithm due to Suurballe [16,17], which is described below, where $l_i=\sum_{e\in P_i}\omega_2(e)$ is the weighted sum of the links in P_i, $1\le i\le K$.

`Find_K_Paths` $(G_{s,t}^{\text{edge}}, s', t'', \omega_2, K)$
begin
$E_K=\emptyset$;/* the link set of the K paths from s' to t''*/
for $i=1$ **to** K **do**
  $E_{\text{res}}=\{\langle u,v\rangle:\langle v,u\rangle\in E_K\}$;/*redirecting all edges in E_K*/
  find a shortest path P_i in $G_{s,t}^{i}$ from s' to t'', where $G_{s,t}^{i}=(V',E'\cup E_{\text{res}}-E_K)$;
  $E_{\text{insec}}=\{\langle u,v\rangle,\langle v,u\rangle:\langle u,v\rangle\in E_K \& \langle v,u\rangle\in E(P_i)\}$;
  $E_K=E_K\cup E(P_i)-E_{\text{insec}}$
  endfor
$G(V(E_K),E_K)$ is a subgraph of $G_{s,t}^{\text{edge}}$ containing the K edge-disjoint paths from s' to t''.
end.

Having constructed $G(V(E_K),E_K)$, the K edge-disjoint paths from s' to t'' can be easily found because the incoming and outgoing degrees of each node except s' and t'' are 1 s. It has been shown that the weighted sum of the links in the K paths by algorithm `Find_K_Paths` is minimum [16] and each such a path can be transformed to a semilightpath in G from s to t. Thus, we have Lemma 1.

Lemma 1. *Let $P_1,P_2,\ldots,P_K$ be the K edge-disjoint paths in $G_{s,t}^{\text{edge}}$ from s' to t'' delivered by the proposed algorithm. Then, the corresponding K semilightpaths $PH_1,PH_2,\ldots,PH_K$ in G are edge-disjoint.*

Proof. If PH_i and PH_j do not share any physical optical link, they are edge-disjoint. Now, assume that PH_i and PH_j share a physical optical link $e=\langle u,v\rangle$. Let e_a be the corresponding link of e in P_i and assigned wavelength λ_a, let e_c be the corresponding link of e in P_j and assigned wavelength λ_b. Then, $\lambda_a\ne\lambda_b$. We show this by contradiction. Assume that $\lambda_a=\lambda_b$. Let e_b be the immediate predecessor of e_a in P_i and e_d the immediate predecessor of e_c in P_j. Then, the two nodes tail(e_a) and tail(e_c) are the same node in $G_{s,t}^{\text{edge}}$ and the incoming degree of tail(e_a) is one. Thus, $e_b=e_d$. In other words, P_i and P_j share a link e_b, which contradicts the assumption that they are edge-disjoint. Therefore, $\lambda_a\ne\lambda_b$.

If PH_i and PH_j do not share any node in G, it is obvious that they are edge-disjoint. Otherwise, assume that they share a node $v\in V$, which is illustrated in Fig. 5. where e_a and e_c are the links entering v and e_b and e_d are the links

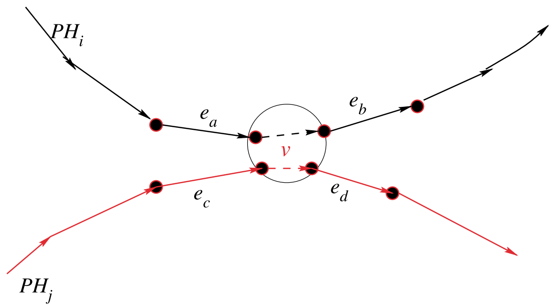

Fig. 5. PH_i and PH_j share a node $v \in V$.

leaving from v. Assume that e_a and e_b are the links in PH_i and e_c and e_d are the links in PH_j.

If e_a and e_c are the same physical optical link, then it is obvious that the wavelengths assigned to them are different; otherwise, e_a and e_c are the two different physical optical links. and the wavelengths assigned to them are different too, which is shown below.

Assume that they are assigned with the same wavelength λ. Let e'_a and e'_c be their corresponding links in P_i and P_j, respectively. Following the construction of $G^{\text{edge}}_{s,t}$, head$(e'_a)=$ head(e'_c) and the outgoing degree of node head(e'_a) is one. Let e'_x be the immediate successor of e'_a in P_i and e'_y the immediate successor of e'_c in P_j. Then, e'_x and e'_y are the the same link in $G^{\text{edge}}_{s,t}$ due to that head$(e'_a)=$ head(e'_c) and the outgoing degree of head(e'_a) is one, i.e., P_i and P_j share one link e'_x at least. This contradicts that P_i and P_j are edge-disjoint. So, e_a and e_c must be assigned with different wavelengths.

If e_b and e_d are the same physical optical link, then the wavelengths assigned to them are different; otherwise, e_b and e_d are two different physical optical links, and the wavelengths assigned to them are different too, which is shown as follows. Assume that both e_b and e_d are assigned with the same wavelength λ. Let e'_b and e'_d be their corresponding links in P_i and P_j, respectively. Then, tail$(e'_b)=$ tail(e'_d) and the incoming degree of tail(e'_b) in $G^{\text{edge}}_{s,t}$ is one. Let e'_x be the immediate predecessor of e'_b in P_i and e'_y the immediate predecessor of e'_d in P_j. Then, $e'_x=e'_y$ due to that tail$(e'_b)=$ tail(e'_d) and the incoming degree of tail(e'_b) is one, which means that P_i and P_j share one link e'_b at least. This contradicts that P_i and P_j are edge-disjoint. Therefore, any two PH_i and PH_j are edge-disjoint with $i \neq j$. □

Lemma 2. *The total cost of the K corresponding semilightpaths found by algorithm* `Find_K_Paths` *is minimum.*

Proof. Let $PH_1, PH_2, \ldots, PH_K$ be the K edge-disjoint semilightpaths in G from s to t with the minimization of the total cost of the K paths and M the total cost of these K semilightpaths. Then, following the construction of $G^{\text{edge}}_{s,t}$, there is a corresponding directed path P_i in $G^{\text{edge}}_{s,t}$ from s' to t'' and the weighted sum of the links in P_i equals the cost of PH_i for each PH_i, $1 \leq i \leq K$. In the following we show that P_i and P_j in $G^{\text{edge}}_{s,t}$ are edge-disjoint by contradiction, for any i and j with $i \neq j$, $1 \leq i,j \leq K$. Assume that P_i and P_j share a link $e=\langle x,y \rangle$ derived by a wavelength λ.

If $\langle x,y \rangle$ is a link between a node in G_u and a node in G_v, then there is a corresponding physical optical link $\langle u,v \rangle$ in G. As a result, PH_i and PH_j share the same physical link and use the same wavelength λ. This contradicts the definition of edge-disjoint semilightpaths. Therefore, P_i and P_j do not share link $\langle x,y \rangle$ in $G^{\text{edge}}_{s,t}$. Otherwise, assume that $e=\langle x,y \rangle$ is a link in G_v, and is in one of the three edge sets (i) $e \in \{\langle x^{(1)}, x^{(2)} \rangle : x^{(1)} \in X^1_v, x^{(2)} \in X^2_v\}$; (ii) $e \in \{\langle x^{(2)}, y^{(1)} \rangle : x^{(2)} \in X^2_v, y^{(1)} \in Y^1_v\}$; and (iii) $e \in \{\langle y^{(1)}, y^{(2)} \rangle : y^{(1)} \in Y^1_v, y^{(2)} \in Y^2_v\}$. If e is a type (i) link, then, both PH_i and PH_j use the same link to enter v and the same wavelength is used on the link. This contradicts edge-disjoint requirement between the two paths. Thus, P_i and P_j do not share link $\langle x,y \rangle$ in $G^{\text{edge}}_{s,t}$. If e is a type (ii) link, then tail$(e)=x$ and the incoming degree of x in $G^{\text{edge}}_{s,t}$ is one. Let e' be a link with head$(e')=x$. Then, P_i and P_j share links e and e'. From tail(e'), it can be derived that both PH_i and PH_j use the same link to enter v and the same wavelength is used on the link. This contradicts that they are edge-disjoint semilightpaths. Therefore, P_i and P_j do not share link $\langle x,y \rangle$ in $G^{\text{edge}}_{s,t}$. If e is a type (iii) link, then, both PH_i and PH_j use the same link to leave v and the same wavelength is used on the link. This contradicts that PH_i and PH_j are edge-disjoint semilight-paths. P_i and P_j thus do not share link $\langle x,y \rangle$ in $G^{\text{edge}}_{s,t}$. Therefore, for given K edge-disjoint semilightpaths PH_1,-$PH_2, \ldots, PH_K$, the corresponding K directed paths $P_1, P_2, \ldots, P_K$ in $G^{\text{edge}}_{s,t}$ from s' to t'' are edge-disjoint and the weighted sum of the links in them is equal to the cost sum of the links in the K semilightpaths.

Let $P'_1, P'_2, \ldots, P'_K$ be K edge-disjoint paths from s' to t'' delivered by algorithm `Find_K_Paths` and $PH'_1, PH'_2, \ldots, PH'_K$ the corresponding semilightpaths. Let M' be the weighted sum of the links in the K semilightpaths, then, $M' \leq M$. While it is known that any directed path in $G^{\text{edge}}_{s,t}$ from s' to t'' corresponds to a semilightpath in G from s to t and M is the minimum total cost of such K edge-disjoint semilightpaths, then, $M \leq M'$. Thus, $M = M'$.

Following the above lemmas, we have the theorem below. □

Theorem 1. *Given a WDM network $G(V,E,\Lambda)$ and a pair of nodes s and t, assume that each link e in G is assigned a set $\Lambda(e) \subseteq \Lambda$ of wavelengths, and every node has a wavelength conversion table. There is an algorithm for finding K edge-disjoint semilightpaths in G from s to t such that the cost sum of the links in the K paths is minimized. The algorithm takes $O(kK(kn+m+n\log(kn)))$ time, where $K \geq 2$.*

Proof. The construction of the directed, weighted auxiliary graph $G^{\text{edge}}_{s,t}=(V', E', \omega_2)$ takes $O(k^2n+km)$ time, because the graph contains no more than $O(k^2n+km)$ links and $O(kn)$ nodes. It is well known that finding a shortest path in a directed, weighted graph H takes $O(m'+n'\log n')$ time if the Fibonacci heap technique [7] is employed (see the book by Cormen et al. [6] on page 530), where H contains n'

nodes and m' edges. Note that the algorithm repeatedly constructs a shortest path tree rooted at s' in a directed weighted graph containing no more than $4kn+2$ nodes and $k^2n+km+4k$ links. So, the algorithm can be implemented in $O(kK(kn+m+n\log(kn)))$ time. The theorem then follows. □

4.2. K edge-disjoint semilightpaths with minimizing the cost of a maximum cost path

We now consider finding K edge-disjoint semilightpaths in G from s to t such that the cost of the maximum cost path is minimized. Despite that the first version of the problem is polynomially solvable, the second version of the problem is NP-hard, which is shown below.

Lemma 3. *Given a WDM network $G(V,E,\Lambda)$ and a source s and a destination t, finding K edge-disjoint semilightpaths in G with the minimization of the cost of the maximum cost path is NP-hard, where $K\geq 2$.*

Proof. We consider a special case with $K=2$. Assume that for a given WDM network $G(V,E,\Lambda)$, each physical optical link e is assigned one wavelength $\lambda\in\Lambda$, associated with a weight $w(e,\lambda)$ for each $e\in E$. The switch at each node is allowed to switch any incoming wavelength to any outgoing wavelength (fully switching) and the conversion cost is 0. Then, finding two edge-disjoint semilightpaths in G from s to t with minimizing the cost of the maximum cost path is exactly equivalent to finding two edge-disjoint paths in $G_{s,t}^{\text{edge}}$ from s' to t'' with minimizing the maximum weighted sum of the links in one of the two paths, while this latter problem has been shown to be NP-complete [11]. Therefore, the problem with minimizing the cost of the maximum cost path is NP-hard. □

Since the second version of the problem is NP-hard, we instead focus on finding an approximate solution. We use algorithm `Find_K_Paths` to find K edge-disjoint paths in $G_{s,t}^{\text{edge}}$ from s' to t'' with minimizing the weighted sum of the links in the K paths. As results, the K edge-disjoint semilightpaths are found. Clearly, these K semilightpaths is an approximate solution of the second version of the problem. We now analyze the performance ratio of this approximation algorithm by Lemma 4.

Lemma. **4** *Given a WDM network $G(V,E,\Lambda)$ and a pair of nodes s and t, assume that there are r edge-disjoint semilightpaths from s to t, $1\leq r\leq K$, then the solution delivered by `Find_K_Paths`$(G_{s,t}^{\text{edge}}, s', t'', \omega_2, r)$ is within r times of the optimum if $r\geq 2$.*

Proof. Let P be a directed path in $G_{s,t}^{\text{edge}}$ from v_0 to v_p consisting of links $\langle v_0,v_1\rangle,\langle v_1,v_2\rangle,\ldots,\langle v_{p-2},v_{p-1}\rangle\langle v_{p-1},v_p\rangle$. The weighted sum of the links in P is $\sum_{\langle v_i,v_{i+1}\rangle\in P}^{i=0,\ldots,p-1}\omega_2(v_i,v_{i+1})$, which is *the cost* of P. Assume that there are r edge-disjoint paths $P_1^{(r)},P_2^{(r)},\ldots,P_r^{(r)}$ in $G_{s,t}^{\text{edge}}$ from s' to t'' with minimizing the total weighted sum of the links in the r paths. Let $l_1^{(r)},l_2^{(r)},\ldots,l_r^{(r)}$ be the weighted sums of the links in r paths, respectively. Without loss of generality, assume that $l_1^{(r)}\leq l_2^{(r)}\leq l_3^{(r)}\leq\cdots\leq l_r^{(r)}$. By this assumption that $0<l_i^{(r)}\leq l_j^{(r)}$ if $1\leq i<j\leq r$, then $l_i^{(r)}/l_j^{(r)}\leq 1$. It is obvious that $l_{r-1}^{(r-1)}\leq l_r^{(r-1)}\leq l_r^{(r)}$ and $l_{r-1}^{(r-1)}+\cdots+l_1^{(r-1)}\leq l_{r-1}^{(r)}+\cdots+l_1^{(r)}$.

Let $Q_1^{(r)},Q_2^{(r)},\ldots,Q_r^{(r)}$ be the r edge-disjoint paths in $G_{s,t}^{\text{edge}}$ from s' to t'' with minimizing the maximum cost of a path among the r paths and let $t_1^{(r)},t_2^{(r)},\ldots,t_r^{(r)}$ be the weighted sums of the links in the r paths, respectively. Without loss of generality, assume that $t_1^{(r)}\leq t_2^{(r)}\leq t_3^{(r)}\leq\cdots\leq t_r^{(r)}$. In other words, $Q_r^{(r)}$ is the path with the maximum cost and $t_r^{(r)}=\max\{t_i^{(r)}:1\leq i\leq r\}$. Following the assumption that $0<t_i^{(r)}\leq t_j^{(r)}$ if $1\leq i<j\leq r$, then $t_i^{(r)}/t_j^{(r)}\leq 1$. It is obvious that $\sum_{i=1}^r t_i^{(r)}\geq\sum_{i=1}^r l_i^{(r)}$ and $t_r^{(r)}\leq l_r^{(r)}$. Thus, we have,

$$\frac{l_r^{(r)}}{t_r^{(r)}}=\frac{(l_r^{(r)}+l_{r-1}^{(r)}+\cdots+l_1^{(r)})-(l_{r-1}^{(r)}+\cdots+l_1^{(r)})}{t_r^{(r)}}$$

$$\leq\frac{t_r^{(r)}+t_{r-1}^{(r)}+\cdots+t_1^{(r)}}{t_r^{(r)}}-\frac{l_{r-1}^{(r)}+\cdots+l_1^{(r)}}{t_r^{(r)}}$$

$$\leq r-\frac{l_{r-1}^{(r)}+\ldots+l_1^{(r)}}{t_r^{(r)}}\leq r-\frac{l_{r-1}^{(r)}+\ldots+l_1^{(r)}}{l_r^{(r)}}$$

$$<r,\quad\text{since}\quad\frac{l_{r-1}^{(r)}+\ldots+l_1^{(r)}}{l_r^{(r)}}>0\quad\text{and}\quad r\geq 2.$$

□

Thus, we have Theorem 2

Theorem 2. *Given a WDM network $G(V,E,\Lambda)$ and a pair of nodes s and t, assume that each link e in G is assigned with a set $\Lambda(e)\subseteq\Lambda$ of wavelengths and every node is given a wavelength conversion table. There is an approximation algorithm for finding K edge-disjoint semilightpaths in G from s to t such that the cost of the maximum cost path is minimized. The algorithm takes $O(kK(kn+m+n\log(kn)))$ time, and the solution delivered is within K times of the optimum, where $K\geq 2$.*

Proof. By setting $r=K$ in Lemma 4, the theorem then follows. □

5. Distributed implementation

In this section we address the distributed implementation issues of the proposed algorithms. The basic idea behind is to embed the ideal network $G_{s,t}^{edge}$ into the physical network G and to simulate $G_{s,t}^{edge}$ using G. The detailed implementation is as follows.

For each node $v\in V$ in G, a directed weighted graph $G_v=(X_v^1\cup X_v^2\cup Y_v^1\cup Y_v^2,E_v,\omega_1)$ is constructed. Each physical link $e\in E$ in G serves as the corresponding $|\Lambda(e)|$ links in $G_{s,t}^{\text{edge}}$. As a result, $G_{s,t}^{\text{edge}}$ is constructed and represented distributively, i.e. every node v in G holds the adjacency lists of the nodes in subgraph G_v of $G_{s,t}^{\text{edge}}$. The problem then becomes to find a shortest path in $G_{s,t}^{i}$ (which is defined in algorithm `Find_K_Paths` and $G_{s,t}^{1}=G_{s,t}^{\text{edge}}$) from s' to t'', $1\le i\le K$. The network G is then used to simulate $G_{s,t}^{i}$, i.e. each node in G actually represents a subgraph in $G_{s,t}^{i}$, $1\le i\le K$. While the single-source shortest paths problem is a well studied problem in the distributed environment, there are many efficient algorithms for it including the algorithm by Chandy and Misra [3]. Therefore, we have the following theorem.

Theorem 3. *Given a WDM network $G(V,E,\Lambda)$ and a pair of nodes s and t, assume that every link e in G is assigned a set $\Lambda(e)$ of wavelengths, and every node is given a wavelength conversion table. There is a distributed algorithm for the multiple routing path problem. The communication and time complexities of the distributed implementation of the proposed algorithm are $O(kKm)$ and $O(kKn)$, respectively, on a distributed computational model.*

Proof. Since the local computation is negligible in the distributed computing environment, the construction of $G_{s,t}^{\text{edge}}$ can be done in constant time. G is then used to simulate $G_{s,t}^{i}$ for each i, $1\le i\le K$. That is, each node in G simulates a subgraph in $G_{s,t}^{i}$ containing at most $4k+1$ nodes. It is known that the communication and time complexities for finding a shortest path between two nodes in a directed graph H with n' nodes and m' links are $O(m')$ and $O(n')$, respectively [3]. Therefore, the communication and time complexities of the distributed implementation of the proposed algorithm are $O(kKm)$ and $O(kKn)$, because the links in $\cup E_v$ in $G_{s,t}^{\text{edge}}$ are the *virtual links* that are within the physical nodes in G. By the definition of this model, the communication costs on these links are negligible. □

6. Conclusions

In this paper a multiple routing path problem with different optimization objectives has been defined. Two versions of the problem have been considered. One is to find K edge-disjoint semilightpaths between a pair of nodes such that the total cost of the K paths is minimized. Another is to find K edge-disjoint semilightpaths between a pair of nodes such that the cost of the maximum cost path is minimized. An efficient algorithm for the first version has been presented and an exact solution is delivered. While the second version of the problem has been shown to be NP-hard, and an approximation algorithm has been proposed, which deliverers a solution within K times of the optimum.

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