The Australian National University Faculty of Engineering & Information Technology

Department of Computer Science

COMP1120-2003-Lab-9

Notes for Laboratory Session 9

1 Objectives

The objective of this session is to give you some practice using mathematical logic; and, hopefully, to give you a chance to begin working on Assignment 3.

2 Laboratory Exercises

None for today; if you finish your Tutorial Exercises early, begin working on Assignment 3.

3 Tutorial Exercises

1. **Topic and Motivation**

   This tutorial work is a brief introduction to the language of mathematical logic. It is difficult to express complex or subtle specifications in English — or other human languages — ambiguities, omissions and contradictions are almost inevitable.

   We want to be able to specify simply and precisely what a computer system is supposed to do. For such tasks the language of mathematical logic is superior to English.

2. **Exercises**

   Starred exercises are extension work. They are optional, and you should only work on them when you have finished all the others.

   (a) Translate each of the following into propositional statement forms:
      i. If the forecast is for rain and I am walking to work, then I'll take an umbrella.
      ii. Either the murderer has left the country or somebody is harbouring him.
      iii. If the murderer has not left the country, then somebody is harbouring him.

   (b) Write out truth tables for the following propositional statement forms:
      i. $((\sim p) \wedge (\sim q))$
      ii. $(p \rightarrow (q \rightarrow r))$
      iii. $((p \wedge q) \rightarrow r)$

   (c) Use a truth table to show that each of the following pairs of propositional statement forms are equivalent.
      i. $(\sim(\sim p))$ and p
      ii. $(\sim(p \wedge q))$ and $((\sim p) \vee (\sim q))$
      iii. $(p \rightarrow q)$ and $((\sim q) \rightarrow (\sim p))$
      iv. $(p \rightarrow q)$ and $((\sim p) \vee q)$

(d) Use truth tables and classic logic to solve each of the following logic puzzles:

i. Suppose the following two statements are true:
(1) I love Betty or I love Jane.
(2) If I love Betty then I love Jane.
Does it necessarily follow that I love Betty?
Does it necessarily follow that I love Jane?
Does it necessarily follow that I do not love Betty?
Does it necessarily follow that I do not love Jane?

ii. Suppose someone asks me,'Is it really true that if you love Betty then you also love Jane?'
I reply, 'If it is true, then I love Betty.'
Does it follow that I love Betty?
Does it follow that I love Jane?

iii. This time we are given two girls, Eva and Margaret.
Someone asks me, 'Is it really true that if you love Eva then you also love Margaret?'
I reply, 'If it is true, then I love Eva, and if I love Eva, then it is true.'
Which girl do I necessarily love?

iv. This time we are given three girls, Sue, Marcia, and Dianne.
Suppose that the following facts are given:
(1) I love at least one of the three girls.
(2) If I love Sue but not Dianne, then I also love Marcia.
(3) I either love both Dianne and Marcia or I love neither one.
(4) If I love Dianne, then I also love Sue.
Which of the girls do I love?

(e) * Express each of the following statements in predicate calculus.

i. All the children of South Park love Cheesy Poofs.

ii. If the underpants gnomes exist then Tweak is not insane.

iii. Everybody loves somebody.

iv. You can fool all of the people some of the time, and you can fool some of the people all of the time, but you can't fool all of the people all of the time.

(f) * Suppose a is an array of integers, what does the following property say about a?

$$\forall i \in Z, \forall j \in Z, (a.\texttt{lower} \leq i \leq j \leq a.\texttt{upper}) \rightarrow (a.\texttt{item}(i) \leq a.\texttt{item}(j))$$

(g) * Suppose a and b are arrays, write a statement in predicate calculus that states that a and b have the same bounds and the same contents.

(h) * Suppose s and t are both strings, write a statement in predicate calculus that states that t is the reverse of s. Would an empty string have a reverse?

(i) * Suppose we wished to represent a set of integers with an array whereby we would consider some integer i to be an element of the set represented by the array a if there was some valid index j into a such that $a.\texttt{item}(j) = i$. Formally restate in the predicate calculus the condition for i to be considered an element of a.