Mathematics in Software Engineering

Lecture Plan:

- Mathematics and Software
- Reasoning about Software
- Speaking Mathematics
- Puzzles

What is Mathematics?

It is not easy to say what mathematics is, but “I know it when I see it” is the most likely response of anyone to whom this question is out.

“What Is Mathematics?”

John D Barrow

Mathematics is the science which draws necessary conclusions.

Benjamin Peirce

Mathematical Models

Using mathematical models. . .

- allows reasoning about the product . . . *before* and after it is built
  - does it satisfy customer requirements?
  - does it behave reasonably?
  - does it interact ok with other products?
- is a form of abstraction
  - gives *some* details
  - leaves out others

Mathematics and Software

We use maths in Software Engineering. . .

- to specify the system
- to model properties of system components
- to help check correctness
- to reason about the system

Mathematics in Software Engineering is a Good Thing

Reasoning from Logical Assumptions

A *calculus* of propositions is a process by which we reason or compute using symbols. For a 'calculus' we need 'axioms' or assumed facts and 'rules' for the manipulation of axioms to produce new facts.

Discrete Mathematics for Computing
John E Munro

We have stressed repeatedly, though not always explicitly, that mathematics owes its unique position to its adherence to the axiomatic method.
As we have pointed out, this method consists of starting with a few statements (axioms) whose truth is taken for granted then deriving other statements from them by applying the rules of logic *alone*.

Mathematics and Logic
Mark Kac & Stanislaw Ulam

What is an Axiom?

axiom *n.* **1.** a recognised truth. **2.** an established and universally accepted principle or rule. **3.** *Logic, Maths.,etc.* a proposition which is assumed without proof for the sake of studying the consequences that follow from it. [L *axioma* from Gk: a requisite]

Macquarie Dictionary

- Axioms are fundamental statements that we accept as true
- Axioms are a core set of properties
- Axioms are *obviously* true and cannot be proven
- Axioms may be used to provide the basis for a formal system

Axioms in Software Engineering

We use axioms in Software Engineering. . .

- to define a formal system in a universal language (mathematics)
- to aid understanding of a software system, without having to study the code
- to specify what a routine/class/program/system should do
- to guide us towards appropriate require and ensure clauses
- to help us to decide if routine/class/program/system can be replaced by another

Why Be Formal?

Designing such software requires great care and attention to detail. So the CAA [Civil Aviation Authority], along with Praxis, a British software company, chose to use a technique known as formal methods during its design process. This relies on mathematical models of software languages rather than a “natural” language such as English to describe what the software will do. These mathematical models can then be used to test the design.

According to Hall, the Praxis designers found that mathematics helped by forcing them to ask lots of questions about how the users would interact with the final system, as they tried to convert the user’s requirements into logic equations for the formal specification.

’’Why Be Formal”
Annabelle McIver

Speaking Mathematics

A reasonable argument can be made that the most important form of statement in mathematics is the **universal conditional statement**:

$\forall x$, if P(x) then Q(x).

Familiarity with statements of this from is essential if you are to learn to speak mathematics.

Discrete Mathematics with Applications
Susannah S Epp

Informal Statement Calculus

Logic, or at least logical mathematics, consists of deduction. We shall examine the rules of deduction making use of the precision which characterises a mathematical approach. In doing this, if we are to have any precision at all we must make our language unambiguous, and the standard mathematical way of doing this is to introduce a symbolic language, with the symbols having precisely stated meaning and uses.

Logic for Mathematicians

A G Hamilton

- Negation NOT $\sim$
- Conjunction AND $\wedge$
- Disjunction OR $\vee$
- Conditional IMPLIES $\rightarrow$
- Biconditional IFF $\iff$

Love and Logic

Suppose the following two statements are true:

(1) I love Betty or I love Jane.

(2) If I love Betty then I love Jane.

Does it necessarily follow that I love Betty?

Does it necessarily follow that I love Jane?

What Is the Name of This Book?

Raymond Smullyan

Does it necessarily follow that I do not love Betty?

Does it necessarily follow that I do not love Jane?

Suggested Reading

From the COMP1110 Reading Brick:

- **Why be Formal**

  McIver, Annabelle (1995) “Why Be Formal” *New Scientist* 26 August 1995, pp 34–38

- **Propositional Calculus**

  Hamilton, A G (1998) *Logic for Mathematicians*, Revised Edition, Cambridge University Press, Cambridge

  Chapter 1 “Informal statement calculus” pp 1–18

- **Predicate Calculus**

  Epp, Susanna S (1995) *Discrete Mathematics with Applications*, 2nd Edition, PWS, Boston

  Chapter 2 “The Logic of Quantified Statements” pp 975–111