ANU College of Engineering and Computer Science
School of Computer Science

COMP1140 – 2009

Assignment 2: Propositional Logic (20 marks)

Due date: Wednesday 21 October
Late penalty: 20% per day

No programming is needed for this assignment. Either drop your assignment into the COMP1140 box on the ground floor of the CSIT Building or send by email to jochen.renz@anu.edu.au. Neat handwriting is acceptable.

1. **(6 marks)** Given the truth tables of the propositional formulas ϕ, ψ, ρ:

a	b	c	d	ϕ	ψ	ρ
0	0	0	0	0	1	0
0	0	0	1	0	1	0
0	0	1	0	1	1	1
0	0	1	1	0	0	1
0	1	0	0	0	0	0
0	1	0	1	1	1	1
0	1	1	0	0	1	0
0	1	1	1	1	0	1
1	0	0	0	0	1	0
1	0	0	1	0	1	1
1	0	1	0	1	1	1
1	0	1	1	0	1	0
1	1	0	0	0	0	0
1	1	0	1	1	0	0
1	1	1	0	1	1	0
1	1	1	1	1	1	0

   (a) **(3 marks)** Write the formulas in DNF using only prime implicants.

   (b) **(3 marks)** Write the formulas in CNF. Use each of the following methods at least once: (1) Karnaugh Maps, (2) directly from the Truth Table, (3) by converting the DNF into CNF using equivalence transformations.

2. **(6 marks)** A *clique* (complete subgraph) of a graph $G = (N, E)$ (N: nodes, E: edges) is a subgraph G' of G such that there is an edge $(n_i, n_j) \in N$ for all nodes n_i, n_j of G'.

   CLIQUE is the problem of deciding whether a graph G has a clique of size k. The problem is NP-hard, which can be proven by reducing 3SAT to CLIQUE using the following transformation:

   Let ϕ be an instance of 3SAT with k clauses. $\phi = (a_1 \vee b_1 \vee c_1) \wedge (a_2 \vee b_2 \vee c_2) \wedge \ldots \wedge (a_k \vee b_k \vee c_k)$. We construct an instance G of CLIQUE as follows:

   Each literal in each clause corresponds to a node in G. The nodes are organized into k groups of 3 nodes each called triples. There is an edge between all pairs of nodes except between:

   - two nodes in the same triple.
   - two nodes corresponding to opposing literals.

   (a) **(4 marks)** Show that ϕ is satisfiable if and only if G has a clique of size k.

   (b) **(2 marks)** Show that CLIQUE is NP-complete.

3. **(8 marks)** Sudoku can be expressed as a propositional formula where the propositional atoms $s_{i,j,k}$ for $1 \leq i, j, k \leq 9$ correspond to whether a number i is in the square in the j-th row and the k-the column.

   Write the following rules of Sudoku as a propositional formula:

   (a) **(2 marks)** Each square contains at most one of the numbers 1 to 9.

   (b) **(2 marks)** Each row contains each of the numbers 1 to 9 at least once.

   (c) **(4 marks)** Use the Resolution method to show that the above two rules guarantee that no two numbers can be in the same square. Note: If necessary, you can sketch the solution by using a smaller Sudoku grid (not smaller than 3x3, please!).