



## Binary

- base 2: 0,1 (true/false) (on/off)
- ...  $2^3$   $2^2$   $2^1$   $2^0$  .  $2^{-1}$   $2^{-2}$   $2^{-3}$  ...  
... 8 4 2 1 . 1/2 1/4 1/8 ...
- binary → decimal:
  - $1001_2 = 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$   
=  $1 \times 8 + 0 \times 4 + 0 \times 2 + 1 \times 1$   
=  $9_{10}$
  - $1101_2 = x_{10}$  ?
- decimal → binary
  - $19_{10} = ?_2$   
 $19/2 = 9$  remainder 1  
 $9/2 = 4$  remainder 1  
 $4/2 = 2$  remainder 0  
 $2/2 = 1$  remainder 0  
 $1/2 = 0$  remainder 1  
 $\rightarrow 19_{10} = 10011_2$
  - $42_{10} = x_2$  ?

## Negative Integers

- positive numbers (one byte):
 

0000 0000	0
0000 0001	1
0000 0010	2
...	...
1111 1111 <sub>2</sub>	255 <sub>10</sub>
- how can we represent negative numbers? e.g.  $-42_{10}$  ?  
 $42_{10} = 0001\ 0101_2$ , but the negative?
  - reserve one bit for sign (highest bit, most significant bit)
  - positive numbers: as above but largest is:
 

0 111 1111 <sub>2</sub>	127 <sub>10</sub>
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  - negative numbers:
 

1 000 0000	-0
1 000 0001	-1
...	...
1 111 1111 <sub>2</sub>	-127 <sub>10</sub>
  - problems: two zeros (!), addition is not simple:  $4 + (-1) = -5$

## Binary Integer Addition and Subtraction

- as per decimal from right to left, propagating 'carries'
- addition:
 

11010 <sub>2</sub>	+	26 <sub>10</sub>	+
<u>01011<sub>2</sub></u>	=	<u>11<sub>10</sub></u>	=
100101 <sub>2</sub>		37 <sub>10</sub>	
- subtraction:
 

10101 <sub>2</sub>	-	21 <sub>10</sub>	-
<u>01011<sub>2</sub></u>	=	<u>11<sub>10</sub></u>	=
01010 <sub>2</sub>		10 <sub>10</sub>	

## Two's Complement Representation of Signed Integers

- most common system
  - only one zero; left-most bit indicates sign
  - normal binary addition gives correct result
- rule: to negate a number, flip the bits to the left of the right-most 1
  - equivalently: flip all the bits and add 1 to the result
- examples:
 

0000 0101	5	1111 1000	-8
1111 1011	-5	0000 1000	8
- addition and subtraction (for 4-bit arithmetic: discard 5th bit, if any):
 

0100	4 +	1101	-3 +
<u>1111</u>	-1 =	<u>1110</u>	-2 =
10011	3	11011	-5

  

0111	7 +	1010	-6 +
<u>0010</u>	2 =	<u>1010</u>	-6 =
1001	-7 (or 9?)	10100	4 (or -12?)

