Relational Databases - Comp2400 / Comp6240

Lecture 5: Sets, Products, Relations, Functions

Maths is more than numbers

The mathematical ideas that relational databases are built on.

- Sets, indexed sets
- Products
  (a more general definition than the usual one)
- Relations
- Functions

This will prepare us for some relational algebra, and normalisation, which is largely about "functional dependencies".

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- things can not be in the set more than once: $\{1, 1\} = \{1\}$

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subtraction $A - B$ is the elements from A that are *not* in B

Exercise: Set operations

Let $A = \{1, 2, 3\}$ and $B = \{true, false\}$.
Which of the following are correct?

1. $\{2\} \subseteq A$
2. $2 \in B$
3. $\{2, 3\} \in (A \cup B)$
4. $2 \in (A \cap B)$
5. $2 \in (A - \{1, 3\})$

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- the same thing can be in a pair twice:
  $(1, 1)$ is an ordered pair
- pairs are 2-tuples, we can have *n*-tuples for any positive integer *n*

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We say that A is *indexed by* the positive integers $\mathbb{Z}^+$.
(do not confuse this with indexed files in computer systems)

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The following slightly more sophisticated definition of relation accounts for both.

It also yields neat and uniform definitions of the main database ideas.

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What is the index set in this example?

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Problem? What about $(5, 5, 5)$?

No problem: $A = \{5\}$, $A_1 = A_2 = A_3 = 5$.
The same element can have several names.

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eg.

$$\{Sydney, Paris\} \times \{2, 4, 11\} = \\ \{(Sydney, 2), (Sydney, 4), (Sydney, 11), \\ (Paris, 2), (Paris, 4), (Paris, 11)\}$$

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Products in queries: `select * from A, B;`

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eg. Let $A_{city} = \{Sydney, Paris\}$ and $A_{custID} = \{2, 4, 11\}$, then

$$\prod_{(i \in \{city, custID\})} A_i =$$

$$\{\{city = Sydney, custID = 2\}, \{city = Sydney, custID = 4\}, \{city = Sydney, custID = 11\}, \{city = Paris, custID = 2\}, \{city = Paris, custID = 4\}, \{city = Paris, custID = 11\}\}$$

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$$\begin{aligned} A_1 \times A_2 &= \prod_{i \in \{1,2\}} A_i \\ &= \{\text{sets } \alpha \text{ indexed by I where each } \alpha_i \in A_i\} \\ &= \{\text{sets } \{a_1, a_2\} \text{ where } a_1 \in A_1 \text{ and } a_2 \in A_2\} \\ &= \{\text{tuples } (a_1, a_2) \text{ where } a_1 \in A_1 \text{ and } a_2 \in A_2\} \end{aligned}$$

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- a database schema is an indexed set of relation schemas with some constraints
  (the index set is the relation schema names)

Functions

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- a *function* $A \longrightarrow B$ is a relation $\subseteq A \times B$ which has exactly one pair (a, b) for each $a \in A$
- if f is a function $A \longrightarrow B$ and $(a, b) \in f$ then we say $f(a) = b$

Exercise: functions

Let $A = \{1, 2, 3\}$ and $B = \{true, false\}$.
Which of the following are functions $A \longrightarrow B$

1. $\{(1, true), (1, false)\}$
2. $\{(1, 2), (2, 3), (3, 1)\}$
3. $\{(1, true), (2, false), (3, true), (1, false)\}$
4. $\{(1, true), (2, false), (3, true), (1, true)\}$
5. $\{(1, true), (2, false), (3, true)\}$
6. $\{(1, true), (3, true)\}$

Labs

- Make sure you enrol in a lab group before next week
- Make sure you attend your lab (2% per lab, and important learning experience)
- Make sure you *understand* what you are doing - if stuck ask your tutor for help
- Remember: *no lecture* next Wednesday 6th August