Relational Databases - Comp2400 / Comp6240

Lecture 12: Sets, Products, Relations, Functions

Mathematical catch-up

Help and exercises for the mathematically challenged.

This will prepare us for some relational algebra, and normalisation, which is largely about “functional dependencies”.

Much of todays material is from *Sets and Groups*, J. A. Green, Routledge & Kegan Paul, 1965.

The first couple of chapters of any discrete maths textbook will help.

Set operations and relations

membership we write $x \in \{x, y, z\}$ to say that x is in the set

subset if every member of A is also in B we write $A \subseteq B$

equality two sets are equal if they have the same members

union we write $A \cup B$ for the set containing everything in A and everything in B

intersection we write $A \cap B$ for the set of things that are in both A and B

subtraction $A - B$ is the elements from A that are *not* in B

Membership, Subset

$\mathbb{Z} = \{$ integers$\}$

Which of the following are true?

1. $42 \subseteq \mathbb{Z}$
2. $\{42\} \subseteq \mathbb{Z}$
3. $42 \in \mathbb{Z}$
4. $\{42\} \in \mathbb{Z}$
5. $\{42\} \in \{\{42\}\}$
6. $\{\} \subseteq \mathbb{Z}$

Union, Intersection, Set-Subtraction

Describe the following sets in words:

$$A = \{x \in \mathbb{Z} \mid 2 \leq x\}$$
$$B = \{x \in \mathbb{Z} \mid x \leq 5\}$$

Find $A \cap B$ and $A \cup B$.

And $A - B$.

Product, Set Equality

What is $\{1\} \times \mathbb{Z}$?

What is $(\{1\} \times \mathbb{Z}) \cap (\mathbb{Z} \times \{2\})$?

Does the following equation make sense?
Is it true?

$$(A \times B) \cap (B \times A) = (A \cap B) \times (A \cap B)$$

"Flat" Products and Tuples

Is $(1,(2,3)) = ((1,2),3)$?

Is $A \times (B \times C) = (A \times B) \times C$

(ie, is set-product *associative*?)

Is $(1,2,3) = (1,2,3)$?

What is $A \times B \times C$?

In database theory, we think if product as an operation on any number of sets, not just 2.

Relations and Functions

Describe this set in words $\{(x,y) \in \mathbb{Z} \times \mathbb{Z} \mid x = y\}$.

Is it a relation? Is it a function?

What about $\{(x,y) \in \mathbb{Z} \times \mathbb{Z} \mid x = 1\}$?
And $\{(x,y) \in \mathbb{Z} \times \mathbb{Z} \mid y = 1\}$?

Is this true?

$$\{(x,y) \in \mathbb{Z} \times \mathbb{Z} \mid x = 1\} = \{1\} \times \mathbb{Z}$$

Function Application

Let $f = \{(x,y) \mid y = 2x\}$ and $g = \{(x,y) \mid y = x + 1\}$.

Then what are

1. $g(4)$
2. $f(g(4))$
3. $g(f(4))$