

Relational Databases - Comp2400 / Comp6240

Lecture 12: Sets, Products, Relations, Functions

Mathematical catch-up

Help and exercises for the mathematically challenged.

This will prepare us for some relational algebra, and normalisation, which is largely about “functional dependencies”.

Much of today's material is from *Sets and Groups*, J. A. Green, Routledge & Kegan Paul, 1965.

The first couple of chapters of any discrete maths textbook will help.

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- subtraction** $A - B$ is the elements from A that are *not* in B

Membership, Subset

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Which of the following are true?

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- 6 $\{\} \subseteq \mathbb{Z}$

Union, Intersection, Set-Subtraction

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And $A - B$.

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Does the following equation make sense?

Is it true?

$$(A \times B) \cap (B \times A) = (A \cap B) \times (A \cap B)$$

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In database theory, we think of product as an operation on any number of sets, not just 2.

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Is this true?

$$\{(x, y) \in \mathbb{Z} \times \mathbb{Z} \mid x = 1\} = \{1\} \times \mathbb{Z}$$

Function Application

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