

Assignment 1

**Natural Deduction and Structural Induction
 SOLUTIONS**

1 Truth Tables in Propositional Logic [3 marks]

Exercise 1.1

Construct truth-tables showing whether the following propositions are valid theorems (tautologies) or not; if not use the truth tables to find truth values for the variables which provide a counterexample.

- (1) [1 mark] $\neg(p \wedge q) \leftrightarrow (\neg p \vee \neg q)$

The proposition is a tautology

p	q	$\neg q$	$\neg p$	$\neg p \vee \neg q$	$p \wedge q$	$\neg(p \wedge q)$	$\neg(p \wedge q) \leftrightarrow (\neg p \vee \neg q)$
T	T	F	F	F	T	F	T
T	F	T	F	T	F	T	T
F	T	F	T	T	F	T	T
F	F	T	T	T	F	T	T

- (2) [2 marks] $(\neg(p \rightarrow q) \vee \neg(q \rightarrow r)) \leftrightarrow (r \rightarrow p)$

The proposition is not a tautology

Counterexamples are $p = q = r = True$, and $p = q = r = False$

p	q	r	$r \rightarrow p$	$q \rightarrow r$	$\neg(q \rightarrow r)$	$p \rightarrow q$	$\neg(p \rightarrow q)$	$\neg(p \rightarrow q) \vee \neg(q \rightarrow r)$	result
T	T	T	T	T	F	T	F	F	F
T	T	F	T	F	T	T	F	T	T
T	F	T	T	T	F	F	T	T	T
T	F	F	T	T	F	F	T	T	T
F	T	T	F	T	F	T	F	F	T
F	T	F	T	F	T	T	F	T	T
F	F	T	F	T	F	T	F	F	T
F	F	F	T	T	F	T	F	F	F

2 Natural Deduction in Propositional Logic [5 marks]

Exercise 2.1

Construct natural deduction proofs for the following derived rules of propositional logic. You may only use the eight introduction and elimination rules discussed in lectures. Do not use truth tables or algebraic laws, or any of the “derived rules” obtained in lectures.

$$(1) \quad [2.5 \text{ marks}] \quad \frac{\neg(p \vee q)}{\neg p \wedge \neg q}$$

1	$\neg(p \vee q)$																																																											
2	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">p</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> p </td> <td></td> </tr> <tr> <td style="padding-right: 10px;">3</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$p \vee q$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> $p \vee q$ </td> <td style="padding-left: 10px;">\vee-I, 2</td> </tr> <tr> <td style="padding-right: 10px;">4</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$(p \vee q) \wedge \neg(p \vee q)$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> $(p \vee q) \wedge \neg(p \vee q)$ </td> <td style="padding-left: 10px;">\wedge-I, 3, 1</td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 10px;"> $\neg p$ </td> <td style="padding-left: 10px;">\neg-I, 2-4</td> </tr> <tr> <td style="padding-right: 10px;">6</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">q</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> q </td> <td></td> </tr> <tr> <td style="padding-right: 10px;">7</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$p \vee q$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> $p \vee q$ </td> <td style="padding-left: 10px;">\vee-I, 6</td> </tr> <tr> <td style="padding-right: 10px;">8</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$(p \vee q) \wedge \neg(p \vee q)$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> $(p \vee q) \wedge \neg(p \vee q)$ </td> <td style="padding-left: 10px;">\wedge-I, 7, 1</td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 10px;"> $\neg q$ </td> <td style="padding-left: 10px;">\neg-I, 6-8</td> </tr> <tr> <td style="padding-right: 10px;">10</td> <td style="border-left: 1px solid black; padding-left: 10px;"> $\neg p \wedge \neg q$ </td> <td style="padding-left: 10px;">\wedge-I, 5, 9</td> </tr> </table> </td></tr></table></td></tr></table></td></tr></table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">p</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">p</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	p			p		3	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$p \vee q$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> $p \vee q$ </td> <td style="padding-left: 10px;">\vee-I, 2</td> </tr> <tr> <td style="padding-right: 10px;">4</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$(p \vee q) \wedge \neg(p \vee q)$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> $(p \vee q) \wedge \neg(p \vee q)$ </td> <td style="padding-left: 10px;">\wedge-I, 3, 1</td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 10px;"> $\neg p$ </td> <td style="padding-left: 10px;">\neg-I, 2-4</td> </tr> <tr> <td style="padding-right: 10px;">6</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">q</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> q </td> <td></td> </tr> <tr> <td style="padding-right: 10px;">7</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$p \vee q$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> $p \vee q$ </td> <td style="padding-left: 10px;">\vee-I, 6</td> </tr> <tr> <td style="padding-right: 10px;">8</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$(p \vee q) \wedge \neg(p \vee q)$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> $(p \vee q) \wedge \neg(p \vee q)$ </td> <td style="padding-left: 10px;">\wedge-I, 7, 1</td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 10px;"> $\neg q$ </td> <td style="padding-left: 10px;">\neg-I, 6-8</td> </tr> <tr> <td style="padding-right: 10px;">10</td> <td style="border-left: 1px solid black; padding-left: 10px;"> $\neg p \wedge \neg q$ </td> <td style="padding-left: 10px;">\wedge-I, 5, 9</td> </tr> </table> </td></tr></table></td></tr></table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$p \vee q$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$p \vee q$</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	$p \vee q$			$p \vee q$	\vee -I, 2	4	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$(p \vee q) \wedge \neg(p \vee q)$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> $(p \vee q) \wedge \neg(p \vee q)$ </td> <td style="padding-left: 10px;">\wedge-I, 3, 1</td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$(p \vee q) \wedge \neg(p \vee q)$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$(p \vee q) \wedge \neg(p \vee q)$</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	$(p \vee q) \wedge \neg(p \vee q)$			$(p \vee q) \wedge \neg(p \vee q)$	\wedge -I, 3, 1	$\neg p$	\neg -I, 2-4	6	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">q</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> q </td> <td></td> </tr> <tr> <td style="padding-right: 10px;">7</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$p \vee q$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> $p \vee q$ </td> <td style="padding-left: 10px;">\vee-I, 6</td> </tr> <tr> <td style="padding-right: 10px;">8</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$(p \vee q) \wedge \neg(p \vee q)$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> $(p \vee q) \wedge \neg(p \vee q)$ </td> <td style="padding-left: 10px;">\wedge-I, 7, 1</td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 10px;"> $\neg q$ </td> <td style="padding-left: 10px;">\neg-I, 6-8</td> </tr> <tr> <td style="padding-right: 10px;">10</td> <td style="border-left: 1px solid black; padding-left: 10px;"> $\neg p \wedge \neg q$ </td> <td style="padding-left: 10px;">\wedge-I, 5, 9</td> </tr> </table> </td></tr></table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">q</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">q</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	q			q		7	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$p \vee q$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> $p \vee q$ </td> <td style="padding-left: 10px;">\vee-I, 6</td> </tr> <tr> <td style="padding-right: 10px;">8</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$(p \vee q) \wedge \neg(p \vee q)$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> $(p \vee q) \wedge \neg(p \vee q)$ </td> <td style="padding-left: 10px;">\wedge-I, 7, 1</td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 10px;"> $\neg q$ </td> <td style="padding-left: 10px;">\neg-I, 6-8</td> </tr> <tr> <td style="padding-right: 10px;">10</td> <td style="border-left: 1px solid black; padding-left: 10px;"> $\neg p \wedge \neg q$ </td> <td style="padding-left: 10px;">\wedge-I, 5, 9</td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$p \vee q$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$p \vee q$</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	$p \vee q$			$p \vee q$	\vee -I, 6	8	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$(p \vee q) \wedge \neg(p \vee q)$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> $(p \vee q) \wedge \neg(p \vee q)$ </td> <td style="padding-left: 10px;">\wedge-I, 7, 1</td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$(p \vee q) \wedge \neg(p \vee q)$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$(p \vee q) \wedge \neg(p \vee q)$</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	$(p \vee q) \wedge \neg(p \vee q)$			$(p \vee q) \wedge \neg(p \vee q)$	\wedge -I, 7, 1	$\neg q$	\neg -I, 6-8	10	$\neg p \wedge \neg q$	\wedge -I, 5, 9
<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">p</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">p</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	p			p																																																							
<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">p</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	p																																																											
p																																																												
3	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$p \vee q$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> $p \vee q$ </td> <td style="padding-left: 10px;">\vee-I, 2</td> </tr> <tr> <td style="padding-right: 10px;">4</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$(p \vee q) \wedge \neg(p \vee q)$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> $(p \vee q) \wedge \neg(p \vee q)$ </td> <td style="padding-left: 10px;">\wedge-I, 3, 1</td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 10px;"> $\neg p$ </td> <td style="padding-left: 10px;">\neg-I, 2-4</td> </tr> <tr> <td style="padding-right: 10px;">6</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">q</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> q </td> <td></td> </tr> <tr> <td style="padding-right: 10px;">7</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$p \vee q$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> $p \vee q$ </td> <td style="padding-left: 10px;">\vee-I, 6</td> </tr> <tr> <td style="padding-right: 10px;">8</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$(p \vee q) \wedge \neg(p \vee q)$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> $(p \vee q) \wedge \neg(p \vee q)$ </td> <td style="padding-left: 10px;">\wedge-I, 7, 1</td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 10px;"> $\neg q$ </td> <td style="padding-left: 10px;">\neg-I, 6-8</td> </tr> <tr> <td style="padding-right: 10px;">10</td> <td style="border-left: 1px solid black; padding-left: 10px;"> $\neg p \wedge \neg q$ </td> <td style="padding-left: 10px;">\wedge-I, 5, 9</td> </tr> </table> </td></tr></table></td></tr></table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$p \vee q$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$p \vee q$</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	$p \vee q$			$p \vee q$	\vee -I, 2	4	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$(p \vee q) \wedge \neg(p \vee q)$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> $(p \vee q) \wedge \neg(p \vee q)$ </td> <td style="padding-left: 10px;">\wedge-I, 3, 1</td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$(p \vee q) \wedge \neg(p \vee q)$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$(p \vee q) \wedge \neg(p \vee q)$</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	$(p \vee q) \wedge \neg(p \vee q)$			$(p \vee q) \wedge \neg(p \vee q)$	\wedge -I, 3, 1	$\neg p$	\neg -I, 2-4	6	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">q</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> q </td> <td></td> </tr> <tr> <td style="padding-right: 10px;">7</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$p \vee q$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> $p \vee q$ </td> <td style="padding-left: 10px;">\vee-I, 6</td> </tr> <tr> <td style="padding-right: 10px;">8</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$(p \vee q) \wedge \neg(p \vee q)$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> $(p \vee q) \wedge \neg(p \vee q)$ </td> <td style="padding-left: 10px;">\wedge-I, 7, 1</td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 10px;"> $\neg q$ </td> <td style="padding-left: 10px;">\neg-I, 6-8</td> </tr> <tr> <td style="padding-right: 10px;">10</td> <td style="border-left: 1px solid black; padding-left: 10px;"> $\neg p \wedge \neg q$ </td> <td style="padding-left: 10px;">\wedge-I, 5, 9</td> </tr> </table> </td></tr></table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">q</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">q</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	q			q		7	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$p \vee q$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> $p \vee q$ </td> <td style="padding-left: 10px;">\vee-I, 6</td> </tr> <tr> <td style="padding-right: 10px;">8</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$(p \vee q) \wedge \neg(p \vee q)$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> $(p \vee q) \wedge \neg(p \vee q)$ </td> <td style="padding-left: 10px;">\wedge-I, 7, 1</td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 10px;"> $\neg q$ </td> <td style="padding-left: 10px;">\neg-I, 6-8</td> </tr> <tr> <td style="padding-right: 10px;">10</td> <td style="border-left: 1px solid black; padding-left: 10px;"> $\neg p \wedge \neg q$ </td> <td style="padding-left: 10px;">\wedge-I, 5, 9</td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$p \vee q$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$p \vee q$</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	$p \vee q$			$p \vee q$	\vee -I, 6	8	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$(p \vee q) \wedge \neg(p \vee q)$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> $(p \vee q) \wedge \neg(p \vee q)$ </td> <td style="padding-left: 10px;">\wedge-I, 7, 1</td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$(p \vee q) \wedge \neg(p \vee q)$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$(p \vee q) \wedge \neg(p \vee q)$</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	$(p \vee q) \wedge \neg(p \vee q)$			$(p \vee q) \wedge \neg(p \vee q)$	\wedge -I, 7, 1	$\neg q$	\neg -I, 6-8	10	$\neg p \wedge \neg q$	\wedge -I, 5, 9									
<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$p \vee q$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$p \vee q$</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	$p \vee q$			$p \vee q$	\vee -I, 2																																																						
<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$p \vee q$</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	$p \vee q$																																																											
$p \vee q$																																																												
4	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$(p \vee q) \wedge \neg(p \vee q)$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> $(p \vee q) \wedge \neg(p \vee q)$ </td> <td style="padding-left: 10px;">\wedge-I, 3, 1</td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$(p \vee q) \wedge \neg(p \vee q)$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$(p \vee q) \wedge \neg(p \vee q)$</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	$(p \vee q) \wedge \neg(p \vee q)$			$(p \vee q) \wedge \neg(p \vee q)$	\wedge -I, 3, 1	$\neg p$	\neg -I, 2-4																																																		
<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$(p \vee q) \wedge \neg(p \vee q)$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$(p \vee q) \wedge \neg(p \vee q)$</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	$(p \vee q) \wedge \neg(p \vee q)$			$(p \vee q) \wedge \neg(p \vee q)$	\wedge -I, 3, 1																																																						
<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$(p \vee q) \wedge \neg(p \vee q)$</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	$(p \vee q) \wedge \neg(p \vee q)$																																																											
$(p \vee q) \wedge \neg(p \vee q)$																																																												
6	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">q</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> q </td> <td></td> </tr> <tr> <td style="padding-right: 10px;">7</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$p \vee q$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> $p \vee q$ </td> <td style="padding-left: 10px;">\vee-I, 6</td> </tr> <tr> <td style="padding-right: 10px;">8</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$(p \vee q) \wedge \neg(p \vee q)$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> $(p \vee q) \wedge \neg(p \vee q)$ </td> <td style="padding-left: 10px;">\wedge-I, 7, 1</td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 10px;"> $\neg q$ </td> <td style="padding-left: 10px;">\neg-I, 6-8</td> </tr> <tr> <td style="padding-right: 10px;">10</td> <td style="border-left: 1px solid black; padding-left: 10px;"> $\neg p \wedge \neg q$ </td> <td style="padding-left: 10px;">\wedge-I, 5, 9</td> </tr> </table> </td></tr></table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">q</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">q</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	q			q		7	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$p \vee q$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> $p \vee q$ </td> <td style="padding-left: 10px;">\vee-I, 6</td> </tr> <tr> <td style="padding-right: 10px;">8</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$(p \vee q) \wedge \neg(p \vee q)$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> $(p \vee q) \wedge \neg(p \vee q)$ </td> <td style="padding-left: 10px;">\wedge-I, 7, 1</td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 10px;"> $\neg q$ </td> <td style="padding-left: 10px;">\neg-I, 6-8</td> </tr> <tr> <td style="padding-right: 10px;">10</td> <td style="border-left: 1px solid black; padding-left: 10px;"> $\neg p \wedge \neg q$ </td> <td style="padding-left: 10px;">\wedge-I, 5, 9</td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$p \vee q$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$p \vee q$</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	$p \vee q$			$p \vee q$	\vee -I, 6	8	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$(p \vee q) \wedge \neg(p \vee q)$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> $(p \vee q) \wedge \neg(p \vee q)$ </td> <td style="padding-left: 10px;">\wedge-I, 7, 1</td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$(p \vee q) \wedge \neg(p \vee q)$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$(p \vee q) \wedge \neg(p \vee q)$</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	$(p \vee q) \wedge \neg(p \vee q)$			$(p \vee q) \wedge \neg(p \vee q)$	\wedge -I, 7, 1	$\neg q$	\neg -I, 6-8	10	$\neg p \wedge \neg q$	\wedge -I, 5, 9																													
<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">q</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">q</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	q			q																																																							
<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">q</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	q																																																											
q																																																												
7	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$p \vee q$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> $p \vee q$ </td> <td style="padding-left: 10px;">\vee-I, 6</td> </tr> <tr> <td style="padding-right: 10px;">8</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$(p \vee q) \wedge \neg(p \vee q)$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> $(p \vee q) \wedge \neg(p \vee q)$ </td> <td style="padding-left: 10px;">\wedge-I, 7, 1</td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 10px;"> $\neg q$ </td> <td style="padding-left: 10px;">\neg-I, 6-8</td> </tr> <tr> <td style="padding-right: 10px;">10</td> <td style="border-left: 1px solid black; padding-left: 10px;"> $\neg p \wedge \neg q$ </td> <td style="padding-left: 10px;">\wedge-I, 5, 9</td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$p \vee q$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$p \vee q$</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	$p \vee q$			$p \vee q$	\vee -I, 6	8	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$(p \vee q) \wedge \neg(p \vee q)$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> $(p \vee q) \wedge \neg(p \vee q)$ </td> <td style="padding-left: 10px;">\wedge-I, 7, 1</td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$(p \vee q) \wedge \neg(p \vee q)$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$(p \vee q) \wedge \neg(p \vee q)$</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	$(p \vee q) \wedge \neg(p \vee q)$			$(p \vee q) \wedge \neg(p \vee q)$	\wedge -I, 7, 1	$\neg q$	\neg -I, 6-8	10	$\neg p \wedge \neg q$	\wedge -I, 5, 9																																						
<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$p \vee q$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$p \vee q$</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	$p \vee q$			$p \vee q$	\vee -I, 6																																																						
<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$p \vee q$</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	$p \vee q$																																																											
$p \vee q$																																																												
8	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$(p \vee q) \wedge \neg(p \vee q)$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> $(p \vee q) \wedge \neg(p \vee q)$ </td> <td style="padding-left: 10px;">\wedge-I, 7, 1</td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$(p \vee q) \wedge \neg(p \vee q)$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$(p \vee q) \wedge \neg(p \vee q)$</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	$(p \vee q) \wedge \neg(p \vee q)$			$(p \vee q) \wedge \neg(p \vee q)$	\wedge -I, 7, 1	$\neg q$	\neg -I, 6-8																																																		
<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$(p \vee q) \wedge \neg(p \vee q)$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$(p \vee q) \wedge \neg(p \vee q)$</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	$(p \vee q) \wedge \neg(p \vee q)$			$(p \vee q) \wedge \neg(p \vee q)$	\wedge -I, 7, 1																																																						
<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$(p \vee q) \wedge \neg(p \vee q)$</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	$(p \vee q) \wedge \neg(p \vee q)$																																																											
$(p \vee q) \wedge \neg(p \vee q)$																																																												
10	$\neg p \wedge \neg q$	\wedge -I, 5, 9																																																										

$$(2) \quad [2.5 \text{ marks}] \quad \frac{p \rightarrow (q \vee r) \quad q \rightarrow r}{p \rightarrow r}$$

1	$p \rightarrow (q \vee r)$																																																									
2	$q \rightarrow r$																																																									
3	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">p</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> p </td> <td></td> </tr> <tr> <td style="padding-right: 10px;">4</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$q \vee r$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> $q \vee r$ </td> <td style="padding-left: 10px;">\rightarrow-E, 1, 3</td> </tr> <tr> <td style="padding-right: 10px;">5</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">q</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> q </td> <td></td> </tr> <tr> <td style="padding-right: 10px;">6</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> r </td> <td style="padding-left: 10px;">\rightarrow-E, 2, 5</td> </tr> <tr> <td style="padding-right: 10px;">7</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> r </td> <td></td> </tr> <tr> <td style="padding-right: 10px;">8</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> r </td> <td style="padding-left: 10px;">R, 7</td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 10px;"> r </td> <td style="padding-left: 10px;">\vee-E, 4, 5-6, 7-8</td> </tr> <tr> <td style="padding-right: 10px;">10</td> <td style="border-left: 1px solid black; padding-left: 10px;"> $p \rightarrow r$ </td> <td style="padding-left: 10px;">\rightarrow-I, 3-9</td> </tr> </table> </td></tr></table></td></tr></table></td></tr></table></td></tr></table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">p</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">p</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	p			p		4	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$q \vee r$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> $q \vee r$ </td> <td style="padding-left: 10px;">\rightarrow-E, 1, 3</td> </tr> <tr> <td style="padding-right: 10px;">5</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">q</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> q </td> <td></td> </tr> <tr> <td style="padding-right: 10px;">6</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> r </td> <td style="padding-left: 10px;">\rightarrow-E, 2, 5</td> </tr> <tr> <td style="padding-right: 10px;">7</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> r </td> <td></td> </tr> <tr> <td style="padding-right: 10px;">8</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> r </td> <td style="padding-left: 10px;">R, 7</td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 10px;"> r </td> <td style="padding-left: 10px;">\vee-E, 4, 5-6, 7-8</td> </tr> <tr> <td style="padding-right: 10px;">10</td> <td style="border-left: 1px solid black; padding-left: 10px;"> $p \rightarrow r$ </td> <td style="padding-left: 10px;">\rightarrow-I, 3-9</td> </tr> </table> </td></tr></table></td></tr></table></td></tr></table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$q \vee r$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$q \vee r$</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	$q \vee r$			$q \vee r$	\rightarrow -E, 1, 3	5	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">q</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> q </td> <td></td> </tr> <tr> <td style="padding-right: 10px;">6</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> r </td> <td style="padding-left: 10px;">\rightarrow-E, 2, 5</td> </tr> <tr> <td style="padding-right: 10px;">7</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> r </td> <td></td> </tr> <tr> <td style="padding-right: 10px;">8</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> r </td> <td style="padding-left: 10px;">R, 7</td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 10px;"> r </td> <td style="padding-left: 10px;">\vee-E, 4, 5-6, 7-8</td> </tr> <tr> <td style="padding-right: 10px;">10</td> <td style="border-left: 1px solid black; padding-left: 10px;"> $p \rightarrow r$ </td> <td style="padding-left: 10px;">\rightarrow-I, 3-9</td> </tr> </table> </td></tr></table></td></tr></table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">q</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">q</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	q			q		6	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> r </td> <td style="padding-left: 10px;">\rightarrow-E, 2, 5</td> </tr> <tr> <td style="padding-right: 10px;">7</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> r </td> <td></td> </tr> <tr> <td style="padding-right: 10px;">8</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> r </td> <td style="padding-left: 10px;">R, 7</td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 10px;"> r </td> <td style="padding-left: 10px;">\vee-E, 4, 5-6, 7-8</td> </tr> <tr> <td style="padding-right: 10px;">10</td> <td style="border-left: 1px solid black; padding-left: 10px;"> $p \rightarrow r$ </td> <td style="padding-left: 10px;">\rightarrow-I, 3-9</td> </tr> </table> </td></tr></table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	r			r	\rightarrow -E, 2, 5	7	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> r </td> <td></td> </tr> <tr> <td style="padding-right: 10px;">8</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> r </td> <td style="padding-left: 10px;">R, 7</td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 10px;"> r </td> <td style="padding-left: 10px;">\vee-E, 4, 5-6, 7-8</td> </tr> <tr> <td style="padding-right: 10px;">10</td> <td style="border-left: 1px solid black; padding-left: 10px;"> $p \rightarrow r$ </td> <td style="padding-left: 10px;">\rightarrow-I, 3-9</td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	r			r		8	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> r </td> <td style="padding-left: 10px;">R, 7</td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	r			r	R, 7	r	\vee -E, 4, 5-6, 7-8	10	$p \rightarrow r$	\rightarrow -I, 3-9
<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">p</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">p</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	p			p																																																					
<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">p</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	p																																																									
p																																																										
4	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$q \vee r$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> $q \vee r$ </td> <td style="padding-left: 10px;">\rightarrow-E, 1, 3</td> </tr> <tr> <td style="padding-right: 10px;">5</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">q</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> q </td> <td></td> </tr> <tr> <td style="padding-right: 10px;">6</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> r </td> <td style="padding-left: 10px;">\rightarrow-E, 2, 5</td> </tr> <tr> <td style="padding-right: 10px;">7</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> r </td> <td></td> </tr> <tr> <td style="padding-right: 10px;">8</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> r </td> <td style="padding-left: 10px;">R, 7</td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 10px;"> r </td> <td style="padding-left: 10px;">\vee-E, 4, 5-6, 7-8</td> </tr> <tr> <td style="padding-right: 10px;">10</td> <td style="border-left: 1px solid black; padding-left: 10px;"> $p \rightarrow r$ </td> <td style="padding-left: 10px;">\rightarrow-I, 3-9</td> </tr> </table> </td></tr></table></td></tr></table></td></tr></table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$q \vee r$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$q \vee r$</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	$q \vee r$			$q \vee r$	\rightarrow -E, 1, 3	5	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">q</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> q </td> <td></td> </tr> <tr> <td style="padding-right: 10px;">6</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> r </td> <td style="padding-left: 10px;">\rightarrow-E, 2, 5</td> </tr> <tr> <td style="padding-right: 10px;">7</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> r </td> <td></td> </tr> <tr> <td style="padding-right: 10px;">8</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> r </td> <td style="padding-left: 10px;">R, 7</td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 10px;"> r </td> <td style="padding-left: 10px;">\vee-E, 4, 5-6, 7-8</td> </tr> <tr> <td style="padding-right: 10px;">10</td> <td style="border-left: 1px solid black; padding-left: 10px;"> $p \rightarrow r$ </td> <td style="padding-left: 10px;">\rightarrow-I, 3-9</td> </tr> </table> </td></tr></table></td></tr></table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">q</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">q</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	q			q		6	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> r </td> <td style="padding-left: 10px;">\rightarrow-E, 2, 5</td> </tr> <tr> <td style="padding-right: 10px;">7</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> r </td> <td></td> </tr> <tr> <td style="padding-right: 10px;">8</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> r </td> <td style="padding-left: 10px;">R, 7</td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 10px;"> r </td> <td style="padding-left: 10px;">\vee-E, 4, 5-6, 7-8</td> </tr> <tr> <td style="padding-right: 10px;">10</td> <td style="border-left: 1px solid black; padding-left: 10px;"> $p \rightarrow r$ </td> <td style="padding-left: 10px;">\rightarrow-I, 3-9</td> </tr> </table> </td></tr></table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	r			r	\rightarrow -E, 2, 5	7	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> r </td> <td></td> </tr> <tr> <td style="padding-right: 10px;">8</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> r </td> <td style="padding-left: 10px;">R, 7</td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 10px;"> r </td> <td style="padding-left: 10px;">\vee-E, 4, 5-6, 7-8</td> </tr> <tr> <td style="padding-right: 10px;">10</td> <td style="border-left: 1px solid black; padding-left: 10px;"> $p \rightarrow r$ </td> <td style="padding-left: 10px;">\rightarrow-I, 3-9</td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	r			r		8	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> r </td> <td style="padding-left: 10px;">R, 7</td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	r			r	R, 7	r	\vee -E, 4, 5-6, 7-8	10	$p \rightarrow r$	\rightarrow -I, 3-9									
<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$q \vee r$</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$q \vee r$</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	$q \vee r$			$q \vee r$	\rightarrow -E, 1, 3																																																				
<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">$q \vee r$</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	$q \vee r$																																																									
$q \vee r$																																																										
5	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">q</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> q </td> <td></td> </tr> <tr> <td style="padding-right: 10px;">6</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> r </td> <td style="padding-left: 10px;">\rightarrow-E, 2, 5</td> </tr> <tr> <td style="padding-right: 10px;">7</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> r </td> <td></td> </tr> <tr> <td style="padding-right: 10px;">8</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> r </td> <td style="padding-left: 10px;">R, 7</td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 10px;"> r </td> <td style="padding-left: 10px;">\vee-E, 4, 5-6, 7-8</td> </tr> <tr> <td style="padding-right: 10px;">10</td> <td style="border-left: 1px solid black; padding-left: 10px;"> $p \rightarrow r$ </td> <td style="padding-left: 10px;">\rightarrow-I, 3-9</td> </tr> </table> </td></tr></table></td></tr></table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">q</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">q</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	q			q		6	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> r </td> <td style="padding-left: 10px;">\rightarrow-E, 2, 5</td> </tr> <tr> <td style="padding-right: 10px;">7</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> r </td> <td></td> </tr> <tr> <td style="padding-right: 10px;">8</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> r </td> <td style="padding-left: 10px;">R, 7</td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 10px;"> r </td> <td style="padding-left: 10px;">\vee-E, 4, 5-6, 7-8</td> </tr> <tr> <td style="padding-right: 10px;">10</td> <td style="border-left: 1px solid black; padding-left: 10px;"> $p \rightarrow r$ </td> <td style="padding-left: 10px;">\rightarrow-I, 3-9</td> </tr> </table> </td></tr></table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	r			r	\rightarrow -E, 2, 5	7	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> r </td> <td></td> </tr> <tr> <td style="padding-right: 10px;">8</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> r </td> <td style="padding-left: 10px;">R, 7</td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 10px;"> r </td> <td style="padding-left: 10px;">\vee-E, 4, 5-6, 7-8</td> </tr> <tr> <td style="padding-right: 10px;">10</td> <td style="border-left: 1px solid black; padding-left: 10px;"> $p \rightarrow r$ </td> <td style="padding-left: 10px;">\rightarrow-I, 3-9</td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	r			r		8	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> r </td> <td style="padding-left: 10px;">R, 7</td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	r			r	R, 7	r	\vee -E, 4, 5-6, 7-8	10	$p \rightarrow r$	\rightarrow -I, 3-9																		
<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">q</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">q</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	q			q																																																					
<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">q</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	q																																																									
q																																																										
6	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> r </td> <td style="padding-left: 10px;">\rightarrow-E, 2, 5</td> </tr> <tr> <td style="padding-right: 10px;">7</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> r </td> <td></td> </tr> <tr> <td style="padding-right: 10px;">8</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> r </td> <td style="padding-left: 10px;">R, 7</td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 10px;"> r </td> <td style="padding-left: 10px;">\vee-E, 4, 5-6, 7-8</td> </tr> <tr> <td style="padding-right: 10px;">10</td> <td style="border-left: 1px solid black; padding-left: 10px;"> $p \rightarrow r$ </td> <td style="padding-left: 10px;">\rightarrow-I, 3-9</td> </tr> </table> </td></tr></table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	r			r	\rightarrow -E, 2, 5	7	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> r </td> <td></td> </tr> <tr> <td style="padding-right: 10px;">8</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> r </td> <td style="padding-left: 10px;">R, 7</td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 10px;"> r </td> <td style="padding-left: 10px;">\vee-E, 4, 5-6, 7-8</td> </tr> <tr> <td style="padding-right: 10px;">10</td> <td style="border-left: 1px solid black; padding-left: 10px;"> $p \rightarrow r$ </td> <td style="padding-left: 10px;">\rightarrow-I, 3-9</td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	r			r		8	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> r </td> <td style="padding-left: 10px;">R, 7</td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	r			r	R, 7	r	\vee -E, 4, 5-6, 7-8	10	$p \rightarrow r$	\rightarrow -I, 3-9																											
<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	r			r	\rightarrow -E, 2, 5																																																				
<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	r																																																									
r																																																										
7	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> r </td> <td></td> </tr> <tr> <td style="padding-right: 10px;">8</td> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> r </td> <td style="padding-left: 10px;">R, 7</td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 10px;"> r </td> <td style="padding-left: 10px;">\vee-E, 4, 5-6, 7-8</td> </tr> <tr> <td style="padding-right: 10px;">10</td> <td style="border-left: 1px solid black; padding-left: 10px;"> $p \rightarrow r$ </td> <td style="padding-left: 10px;">\rightarrow-I, 3-9</td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	r			r		8	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> r </td> <td style="padding-left: 10px;">R, 7</td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	r			r	R, 7	r	\vee -E, 4, 5-6, 7-8	10	$p \rightarrow r$	\rightarrow -I, 3-9																																				
<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	r			r																																																					
<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	r																																																									
r																																																										
8	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black; padding-left: 5px;"> r </td> <td style="padding-left: 10px;">R, 7</td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	r			r	R, 7	r	\vee -E, 4, 5-6, 7-8																																																
<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table> </td> <td style="border-left: 1px solid black;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	r			r	R, 7																																																				
<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">r</td> <td style="border-left: 1px solid black;"></td> </tr> </table>	r																																																									
r																																																										
10	$p \rightarrow r$	\rightarrow -I, 3-9																																																								

3 Natural Deduction in Predicate Logic [3 marks]

Exercise 3.1

Construct natural deduction proof for the following derived rule of propositional logic. You may only use the introduction and elimination rules discussed in lectures. Do not use algebraic laws, or any of the “derived rules” obtained in lectures.

$$\frac{\exists x. P(x) \rightarrow Q}{(\forall x. P(x)) \rightarrow Q}$$

Note the implied bracketing of the top line, it is $\exists x. (P(x) \rightarrow Q)$

1		$\exists x. P(x) \rightarrow Q$	
2		$\forall x. P(x)$	
3			
3			
4			
5			
6			
7		$(\forall x. P(x)) \rightarrow Q$	\rightarrow -I, 2-6

or this alternative proof

1		$\exists x. P(x) \rightarrow Q$	
2			
3			
4			
5			
6			
7		$(\forall x. P(x)) \rightarrow Q$	\exists -E, 1, 2-6

4 Structural Induction [9 marks]

Exercise 4.1 (4 marks)

Here is the usual Haskell definition of a binary tree:

```
data Tree a = Nul | Node a (Tree a) (Tree a)
```

We can count the values stored at the nodes by the Haskell function `count`. We can also count the “leaves”, the empty trees at the bottom, using the Haskell function `countNul`.

```
count Nul = 0 -- (C1)
```

```
count (Node x t1 t2) = 1 + count t1 + count t2 -- (C2)
```

```
countNul Nul = 1 -- (CN1)
```

```
countNul (Node x t1 t2) = countNul t1 + countNul t2 -- (CN2)
```

Prove, using structural induction on trees, that

```
countNul t = 1 + count t
```

Base case: $t = \text{Nul}$

Show that $\text{countNul Nul} = 1 + \text{count Nul}$

Proof:

```
countNul Nul = 1                -- by (CN1)
  = 1 + 0 = 1 + count Nul      -- by (C1)
```

Step case: $t = \text{Node } y \ t1 \ t2$

Show that if

```
countNul t1 = 1 + count t1      -- (IH1)
countNul t2 = 1 + count t2      -- (IH2)
```

then $\text{countNul (Node } y \ t1 \ t2) = 1 + \text{count (Node } y \ t1 \ t2)$

Proof:

```
countNul (Node y t1 t2) = countNul t1 + countNul t2  -- by (CN2)
  = 1 + count t1 + 1 + count t2                       -- by (IH1, IH2)
  = 1 + count (Node y t1 t2)                         -- by (C2)
```

QED

Exercise 4.2 (5 marks)

A tail-recursive function to sum the members of a list is given by `sumTR`. In fact `sumTR xs acc` adds all the members of the list `xs` to the number `acc`

```
sumTR [] acc      = acc                -- (STR1)
sumTR (x:xs) acc = sumTR xs (x + acc) -- (STR2)
```

Prove by induction the following property of `sumTR`

```
sumTR xs (sumTR ys acc) = sumTR (ys ++ xs) acc
```

Hint: you will need to think about whether to prove it by induction on `xs` or `ys`

State clearly what property P is being proved by induction, including any quantifiers needed in the statement of P and in the inductive hypothesis.

Do this by induction on `ys`, need to use

```
P(ys) =  $\forall \text{acc. } \text{sumTR } xs \ (\text{sumTR } ys \ \text{acc}) = \text{sumTR } (ys \ ++ \ xs) \ \text{acc}$ 
```

Base case: $ys = []$

Show $\text{sumTR } xs \ (\text{sumTR } [] \ \text{acc}) = \text{sumTR } ([] \ ++ \ xs) \ \text{acc}$

Proof:

$$\begin{aligned} \text{sumTR } xs \ (\text{sumTR } [] \ acc) &= \text{sumTR } xs \ acc && \text{-- by (STR1)} \\ &= \text{sumTR } ([] \ ++ \ xs) \ acc && \text{-- by (A1)} \end{aligned}$$

Step case: Assume

$$\forall acc. \ \text{sumTR } xs \ (\text{sumTR } as \ acc) = \text{sumTR } (as \ ++ \ xs) \ acc \quad \text{-- (IH)}$$

$$\text{Prove } \quad \forall acc. \ \text{sumTR } xs \ (\text{sumTR } (a:as) \ acc) = \text{sumTR } ((a:as) \ ++ \ xs) \ acc$$

Proof:

$$\begin{aligned} \text{sumTR } xs \ (\text{sumTR } (a:as) \ acc) &= \text{sumTR } xs \ (\text{sumTR } as \ (a+acc)) && \text{-- by (STR2)} \\ &= \text{sumTR } (as \ ++ \ xs) \ (a+acc) && \text{-- by (IH)(*)} \\ &= \text{sumTR } (a:(as \ ++ \ xs)) \ acc && \text{-- by (STR2)} \\ &= \text{sumTR } ((a:as) \ ++ \ xs) \ acc && \text{-- by (A2)} \end{aligned}$$

(*) Note, acc in the IH is instantiated to $a + acc$ when it is used in the proof