

Natural Deduction Rules for the Predicate
Calculus

$$(\forall I) \quad \frac{P(a) \quad (a \text{ arbitrary})}{\forall x. P(x)}$$

$$(\forall E) \quad \frac{\forall x. P(x)}{P(a)}$$

$$(\exists I) \quad \frac{P(a)}{\exists x. P(x)}$$

$$(\exists E) \quad \frac{\exists x. P(x) \quad \begin{array}{c} [P(a)] \\ \vdots \\ q \end{array} \quad (a \text{ arbitrary})}{q \quad (a \text{ not free in } q)}$$

$$(\wedge I) \quad \frac{p \quad q}{p \wedge q}$$

$$(\wedge E) \quad \frac{p \wedge q}{p} \quad \frac{p \wedge q}{q}$$

$$(\vee I) \quad \frac{p}{p \vee q} \quad \frac{q}{q \vee p}$$

$$(\vee E) \quad \frac{\begin{array}{c} [p] \quad [q] \\ \vdots \quad \vdots \\ p \vee q \quad r \quad r \end{array}}{r}$$

$$(\rightarrow I) \quad \frac{\begin{array}{c} [p] \\ \vdots \\ q \end{array}}{p \rightarrow q}$$

$$(\rightarrow E) \quad \frac{p \quad p \rightarrow q}{q}$$

$$(\neg I) \quad \frac{\begin{array}{c} [p] \\ \vdots \\ q \wedge \neg q \end{array}}{\neg p}$$

$$(\neg E) \quad \frac{\begin{array}{c} [\neg p] \\ \vdots \\ q \wedge \neg q \end{array}}{p}$$