

**Natural Deduction Rules for the Propositional
Calculus**

$$\begin{array}{l} (\wedge I) \quad \frac{p \quad q}{p \wedge q} \qquad \qquad \qquad (\wedge E) \quad \frac{p \wedge q}{p} \quad \frac{p \wedge q}{q} \\ \\ (\vee I) \quad \frac{p}{p \vee q} \quad \frac{q}{q \vee p} \qquad \qquad \qquad (\vee E) \quad \frac{\begin{array}{c} [p] \quad [q] \\ \vdots \quad \vdots \\ p \vee q \quad r \quad r \end{array}}{r} \\ \\ (\rightarrow I) \quad \frac{\begin{array}{c} [p] \\ \vdots \\ q \end{array}}{p \rightarrow q} \qquad \qquad \qquad (\rightarrow E) \quad \frac{p \quad p \rightarrow q}{q} \\ \\ (\neg I) \quad \frac{\begin{array}{c} [p] \\ \vdots \\ q \wedge \neg q \end{array}}{\neg p} \qquad \qquad \qquad (\neg E) \quad \frac{\begin{array}{c} [\neg p] \\ \vdots \\ q \wedge \neg q \end{array}}{p} \end{array}$$