

COMP3600 - Algorithms

Design Methodologies: Review and Applications

September 13, 2010

1 Heaps

What is the running time of heapsort on an array A of length n that is initially sorted in increasing order already? What about decreasing order?

2 Hashing and Hash Table

Consider inserting the keys 10, 22, 31, 4, 15, 28, 17, 88, 59 into a hash table of length $m = 11$ using open addressing with the primary hash function $h(k) = k \bmod n$. Illustrate the result of inserting these keys using linear probing, quadratic probing with $c_1 = 1$ and $c_2 = 3$, and double hashing with $h_2(k) = 1 + (k \bmod (m - 1))$.

3 Binary Search Tree

TREE-INSERT(T, z)

```
1   $y \leftarrow \text{NIL}$ 
2   $x \leftarrow \text{root}[T]$ 
3  while  $x \neq \text{NIL}$ 
4      do  $y \leftarrow x$ 
5          if  $\text{key}[z] < \text{key}[x]$ 
6              then  $x \leftarrow \text{left}[x]$ 
7              else  $x \leftarrow \text{right}[x]$ 
8   $p[z] \leftarrow y$ 
9  if  $y = \text{NIL}$ 
10     then  $\text{root}[T] \leftarrow z$ 
11     else if  $\text{key}[z] < \text{key}[y]$ 
12         then  $\text{left}[y] \leftarrow z$ 
13         else  $\text{right}[y] \leftarrow z$ 
```

- Give a recursive version of the TREE-INSERT($T; z$) procedure (see page 294).
- Can you give a procedure to find the predecessor of a key k in a binary search tree? (hint: check procedure TREE-SUCCESSOR(x)).

4 Red-Black Trees

Suppose that a node x is inserted into a red-black tree with RB-INSERT and then immediately deleted with RB-DELETE. Is the resulting red-black tree the same as the initial red-black tree before the key insertion? Justify your answer.

5 Disjoint Sets

CONNECTED-COMPONENTS(G)

```
1  for each vertex  $v \in V[G]$ 
2      do MAKE-SET( $v$ )
3  for each edge  $(u, v) \in E[G]$ 
4      do if FIND-SET( $u$ )  $\neq$  FIND-SET( $v$ )
5          then UNION( $u, v$ )
```

FIND-SET(x)

```
1  if  $x \neq p[x]$ 
2      then  $p[x] \leftarrow$  FIND-SET( $p[x]$ )
3  return  $p[x]$ 
```

Show that after all edges are processed by CONNECTED-COMPONENTS, two vertices are in the same connected component if and only if they are in the same set.