Australian National University
Department of Computer Science

COMP3600/COMP6466 in 2009 – Tutorial 4

Question 1.

A **depth-first** forest classifies the edges in a directed graph into tree edges, back edges, forward edges, and cross edges. A **breadth-first** tree can also be used to classify the edges reachable from the source of the search into the same four categories. Prove that in a breadth-first search of a directed graph, the following properties hold (where $d[v]$ is the distance from the starting vertex to v):

1. There are no forward edges.
2. For each tree edge $\langle u, v \rangle$, we have $d[v] = d[u] + 1$.
3. For each cross edge $\langle u, v \rangle$, we have $d[v] \leq d[u] + 1$.
4. For each back edge $\langle u, v \rangle$, we have $0 \leq d[v] \leq d[u]$.

Question 2.

(a) There are two well-known algorithms, Prim's algorithm and Kruskal's algorithm, for finding minimum spanning trees. What are the essential differences between them?

(b) Assume that the weights assigned to the edges in graph $G(V, E)$ are distinct. Let C be any cycle of G and let e be a maximum-weight edge on C. Show that every minimum spanning tree of $G'(V, E - \{e\})$ is also a minimum spanning tree of G.

Question 3.

Given a directed weighted graph $G(V, E)$ with source s, in which case should the Bellman-Ford algorithm be used instead of Dijkstra's algorithm to solve the single-source shortest paths problem with source s?

Question 4.

Given a directed graph $G(V, E)$, let $d(v_i, v_j)$ be the **distance** in G from v_i to v_j to be the minimum number of edges in a path from v_i to v_j, for $v_i, v_j \in V$. Define the **diameter** D of G to be $D = \max_{v_i, v_j \in V} d(v_i, v_j)$. What is the time complexity for finding the diameter using BFS? repeated squaring?