

# COMP3620/6320: Artificial Intelligence

## Tutorial 2: KRR

### Q1. CNF conversion, Horn KBs, resolution, and DPLL.

You are given the following propositional knowledge base (KB):

- $a \Rightarrow (b \Rightarrow c)$
- $a \Leftrightarrow b$
- $(a \wedge \neg c) \vee (b \wedge \neg c)$

Answer the following questions:

- (a) Are any of these formulae already in CNF? For those that are not, convert them to CNF.
- (b) Is the CNF KB in Horn form? If not, state which clauses are not in Horn form. Can you perform forward-chaining or backward-chaining as a means of inference in this KB?
- (c) Is there a refutation of this clause set via resolution? If so, show it.
- (d) Show how DPLL with unit propagation (but no pure literal elimination) using variable ordering  $a, b, c$  works on this clause set.

### Q2. 2-SAT is Poly-time.

3-SAT is known to be NP-Complete (the input consists of a KB in CNF form where each clause has exactly 3 literals).

Show that there is an algorithm for solving 2-SAT that is polynomial time in the input size (each clause has a maximum of two literals).

### Q3. Translation to FOL and first-order resolution.

Translate the following sentences to first order logic (using only the predicate or function names provided in parenthesis) and use resolution to determine whether the query is entailed:

### Knowledge base (KB):

- (a) A vegetarian pizza is defined as a pizza whose all toppings are vegetables ( $VegetarianPizza(.)$ ,  $Pizza(.)$ ,  $Topping(.,.)$ ,  $Vegetable(.)$ ).
- (b) A non-vegetarian pizza is defined as a pizza with at least one non-vegetable topping ( $NonVegetarianPizza(.)$ ,  $Pizza(.)$ ,  $Topping(.,.)$ ,  $Vegetable(.)$ ).
- (c) There exists a pizza with a Pepperoni topping and Pepperoni is known to not be a vegetable ( $Pizza(.)$ ,  $Topping(.,.)$ ,  $Vegetable(.)$ ).
- (d) There exists a pizza with an Asparagus topping and Asparagus is known to be a vegetable ( $Pizza(.)$ ,  $Topping(.,.)$ ,  $Vegetable(.)$ ).

### Queries:

*Query 1:* Does there exist a vegetarian pizza?

*Query 2:* Does there exist a non-vegetarian pizza?

**Note:** The answer may not be what you initially think it should be... go through the mechanical process of resolution theorem proving to see what can be actually proved. Recall that first-order logic knowledge bases only provide constraints on the world and that these constraints rarely restrict the legal interpretations to a single model.

If humans tend to make additional inferences that cannot be proved via formal theorem proving, one explanation would be that we have additional implicit background assumptions that we use in our reasoning process. If you made different inferences than the theorem prover, how might you formalize the additional background assumptions that led to *your* inferences?