

COMP3630/6363 Assignment 3, 2009

Due date: beginning of lecture, Monday 25 May

Hand-written answers are acceptable if written neatly.

Correct answers might be given less than full credit if they are unnecessarily complicated.

Late penalty: 10% cumulative per day, ignoring the weekend. Not accepted after 1 June.

Question 1

Construct a Turing machine that decides $A = \{0^{2^n} \mid n \geq 0\}$, the set of all strings of 0's whose length is a power of 2. Describe the Turing machine in words, and draw its transition diagram.

Question 2

Either prove or disprove each of the following statements:

- (a) All context-free languages are decidable.
- (b) It's decidable whether a 2-tape Turing machine ever writes a nonblank symbol on its second tape given any input string.
- (c) It's decidable whether a Turing machine has a state that is never entered on any input string.

Question 3

For $n \geq 1$, let E_n be the set of all Turing machines that (1) have two tape symbols 1 and blank, (2) have exactly n states, and (3) halt when starting with a blank tape (i.e., when the input string is the empty string). For $M \in E_n$, let $s(M)$ be the number of steps M takes before halting on empty input. Let $s(n)$ be the largest $s(M)$ for any $M \in E_n$. Show that $s(n)$ is not computable (i.e., there is no Turing machine that will start with n 1's on the tape and halt with $s(n)$ 1's on the tape for all n).