

## COMP3630/6363 Assignment 4, 2009

Due date: Tuesday 9 June

Hand-written answers are acceptable if written neatly.

Correct answers might be given less than full credit if they are unnecessarily complicated.

Late penalty: 10% cumulative per day, ignoring the weekend. Not accepted after 15 June.

### Question 1

- (a) Give a polynomial-time algorithm to solve 2-SAT, the satisfiability of CNF formulas with two literals per clause.
- (b) Show that  $\mathcal{NP}$  is closed under the star operation (i.e.,  $A \in \mathcal{NP} \Rightarrow A^* \in \mathcal{NP}$ ).

### Question 2

We know that it's  $\mathcal{NP}$ -complete to determine whether a graph  $G$  contains a clique of size  $k$ . Show that it's  $\mathcal{NP}$ -complete to determine whether a graph  $G$  contains a clique of size at least  $m/2$ , where  $m$  is the number of nodes of  $G$ .

### Question 3

The game of *Generalized Geography* is played on a directed graph  $G$  with a distinguished “start node”  $s$ . There are two players, P1 and P2. Alternately, with P1 playing first, each player selects a node of  $G$  that must be a successor of the node selected by the other player in the previous move, or a successor of  $s$  in the case of the very first move of the game. A node can only be selected once in the entire game. The player unable to make a move loses. Show that the following problem is PS-complete: Given  $\langle G, s \rangle$ , can P1 win no matter what choices P2 makes?