Due date: Monday, August 14, 2017

Marking scheme: Full marks for a formulation that correctly answers the question and clearly shows the steps to obtain the solution.

**Question 1** (Semantics, 3 + 3 pts). True or false? Prove or find a counterexample:

1. For all formulas $F_1$ and $F_2$ and interpretations $I$, if $(F_1 \rightarrow F_2)$ is valid and $I \not|= F_2$ then $\neg F_1$ is satisfiable.

2. For all formulas $F_1$ and $F_2$, if $(F_1 \rightarrow F_2)$ is satisfiable and $F_1$ is satisfiable then $F_2$ is satisfiable.

**Question 2** (Structural induction, 6 pts). Define recursively a function $\text{trans}$ on formulas as follows, where $P$ is a propositional variable and $F, F_1$ and $F_2$ are formulas:

\[
\begin{align*}
\text{trans}(\top) &= \top \\
\text{trans}(P) &= \neg P \\
\text{trans}(\bot) &= \bot \\
\text{trans}(\neg F) &= \neg \text{trans}(F) \\
\text{trans}(F_1 \circ F_2) &= \text{trans}(F_1) \circ \text{trans}(F_2), \text{ for all } \circ \in \{\land, \lor, \rightarrow, \leftrightarrow\}
\end{align*}
\]

Prove: for all formulas $F$, if $F$ is satisfiable then $\text{trans}(F)$ is satisfiable.

**Question 3** (Tableau calculus, 6 + 6 pts). Consider the following formulas:

(a) $P \lor Q \rightarrow \neg P \land Q$

(b) $(P \rightarrow Q) \rightarrow (\neg Q \rightarrow \neg P)$

For each of these formulas use the Tableau method to prove their validity or invalidity.

**Question 4** (Normal form, 4 pts). Compute the NNF and the CNF of $\neg(P \land (Q \lor \neg R))$.

**Question 5** (Semantic trees and DPLL, 2 + 4 + 4 pts). Consider the clause set \(N = \{P \lor Q, \neg P \lor Q, \neg Q \lor R \lor P, \neg R, \neg P \lor R, \neg R \lor P\}\).

1. Compute $N' = \text{simplify}(N, \neg R)$.

2. Compute a closed semantic tree for $N'$ or for $N$ (your choice).

3. Read off from this semantic tree a resolution refutation.

**Question 6** (Resolution, 6 pts). In a criminal case the following facts have been shown to hold:

1. At least one of the persons $A, B, C$ is guilty.

2. If $A$ is guilty and $B$ is not guilty, then $C$ is guilty.

3. If $B$ is guilty then $C$ is guilty.

Use propositional resolution to prove that one of $A, B, C$ is guilty (who?).

**Question 7** (Semantics, 6 pts). Show that the following infinite clause set is satisfiable (each $P_i$ is a propositional variable). Hint: the compactness theorem might be useful.

\[N = \{P_0\} \cup \{\neg P_i \lor P_{i+2} \mid i \geq 0\} \cup \{\neg P_i \lor \neg P_{i+1} \mid i \geq 0\}\]