Question 1 (Semantics, 3 + 3 pts).

1. Show using truth tables that the following equivalences hold:
   (a) \((F_1 \land F_2) \lor (F_3 \land F_4) \Leftrightarrow (F_1 \lor F_3) \land (F_1 \lor F_4) \land (F_2 \lor F_3) \land (F_2 \lor F_4)\)
   (b) \((F_1 \lor F_2) \land (F_3 \lor F_4) \Leftrightarrow (F_1 \land F_3) \lor (F_1 \land F_4) \lor (F_2 \land F_3) \lor (F_2 \land F_4)\)

2. Use the inductive definition of the semantics to show that if \(I \models (F_1 \rightarrow F_2)\) and \(I \models (F_2 \rightarrow F_3)\) then \(I \models (F_1 \rightarrow F_3)\). Do not use truth tables.

Question 2 (Structural induction, 6 pts). Using structural induction prove the following: for all models \(I\), and all formulae \(F\), either \(I \models F\) or else \(I \models \neg F\).

Question 3 (Tableau calculus, 3 + 3 + 3 pts). Consider the following formulas:

(a) \((P \rightarrow Q) \lor (Q \rightarrow P)\)
(b) \((P \rightarrow Q) \lor (P \rightarrow \neg Q)\)
(c) \((P \rightarrow Q) \rightarrow ((Q \rightarrow R) \rightarrow (P \rightarrow R))\)

For each of these formulas use the Tableau method to prove their validity or invalidity. Give your worked out tableaux. Simply stating valid/invalid is not good enough.

Question 4 (Normal forms, 2 + 2 pts). Give a clearly worked out solution of the NNF and the CNF of \(\neg[(P \rightarrow Q) \rightarrow ((Q \rightarrow R) \rightarrow (P \rightarrow R))]\). Simply giving the answer is not good enough.

Question 5 (Semantic trees and DPLL, 2 + 4 + 4 pts). Consider the clause set
\[ N = \{P \lor Q, \neg Q \lor R \lor P, \neg R, \neg P \lor R, \neg R \lor P, \neg P \lor Q\} \ . \]

1. Compute \(N' = \text{simplify}(N, \neg R)\).
2. Compute a closed semantic tree for \( N' \) or for \( N \) (your choice).

3. Read off from this semantic tree a resolution refutation.

**Question 6** (Resolution, 6 pts). Produce a resolution refutation of the clause set below:

\[
\{ \neg P_1 \lor P_2, \neg P_2 \lor P_3 \lor P_6, \neg P_3 \lor P_4, \neg P_4 \lor P_5, \neg P_5 \lor \neg P_1, P_1, \neg P_6 \}\]

**Question 7** (Tableaux, 3 + 3 pts).

1. Design a tableau calculus based upon the one from your lectures for formulae in CNF. Every superfluous rule will cost you a mark. Every missing rule will cost you a mark. Every rule should be as precise as possible or you will lose another mark.

2. Give a strategy for applying the rules which tries to minimise the number of nodes, and explain your reasoning in at most five sentences. You will be penalised for verbiage as it will indicate a lack of understanding.