Exercise 1. For each of the formulas below, do the following:

1. using the tableau calculus check if it is valid. If it is not go to point 2 of this exercise;
2. using truth tables determine if it is satisfiable. If it is provide both an interpretation making it true and one making it false.

a- \( P \to (Q \to P) \)
b- \((P \to Q) \land \neg P \to \neg Q \)
c- \((\neg P \to \neg Q) \to ((\neg Q \to \neg R) \to (R \to \neg P)) \land (R \land Q) \)
d- \((P \land \neg Q) \lor (\neg P \land Q) \)
e- \(((P \land Q) \to R) \to ((P \to R) \land (Q \to R)) \)
f- \((\neg P \to \neg Q) \to (Q \to P) \)
g- \((\neg Q \land P) \to Q) \)
h- \(((P \to Q) \to P) \to P \)
i- \((\top \to P) \land (P \to \bot) \)
j- \(((P \to Q) \land (\neg P \to Q)) \to Q. \)

Exercise 2. Show that for every formula \( F \) the following holds: \( F \iff (\neg F \to F) \).

Exercise 3. Show that for every formula \( F \), if \( \neg F \Rightarrow \bot \) then \( F \) is valid.

Exercise 4. Show that for every formulas \( F \) and \( G \) the following holds:

\( F \to G \) is valid \iff \( F \land \neg G \) is unsatisfiable.

Exercise 5 (Extended Substitution Theorem). Let \( F \) and \( G \) be formulas such that \( F \iff G \). Show that for every formulas \( H_1, H_2 \) the following holds:

\( H_1 \Rightarrow H_2 \iff H_1 \Rightarrow H_2[F/G] \).

where \( H_2[F/G] \) is the formula obtained after substituting every occurrence of \( G \) in \( H_2 \) by \( F \) (note that the substitution can be vacuous: there might be no such occurrence of \( G \) in \( H_2 \)).