Exercise 1. Consider the following formulae:

- $\forall x. (\neg p(x, x))$
- $\forall x. \exists y. (p(x, y))$
- $\forall x. \forall y. \forall z. ((p(x, y) \land p(y, z)) \rightarrow p(x, z))$
- $\forall x. \forall y. (p(x, y) \rightarrow \exists z. (p(x, z) \land p(z, y)))$

Find a satisfying interpretation for these formulae. Argue why every interpretation satisfying these formulae has an infinite domain.

Exercise 2. We say that a term $t$ is linear if for every variable $x \in \text{var}(t)$ we have that $x$ occurs only once in $t$. Show by induction on the structure of terms that if $t$ is a linear term and $\sigma$ is a substitution such that $\text{varcod}(\sigma) = \emptyset$, then $t\sigma$ is linear.

Exercise 3. Determine if the following multisets of equations are unifiable:

1. $E_1 = \{ p(x, y) = p(z, f(x)), y = f(f(x)), f(z) = f(y) \}$
2. $E_2 = \{ f(x, y) = f(x, t), p(x, t) = p(z, y) \}$
3. $E_3 = \{ g(x, f(y, y), t) = g(y, x, y), p(t) = p(y), x = g(t, t, t) \}$
4. $E_4 = \{ f(h(y), x) = f(z, t), g(z) = g(h(y)), f(h(z), t) = f(h(h(y)), y) \}$

Exercise 4. Using the resolution calculus show that the following set clauses are unsatisfiable:

1. $S_1 = \{ \neg p_0(x) \lor p_1(f(z)), p_2(y) \lor \neg p_1(y), \neg p_2(f(x)) \lor \neg p_0(x), p_0(f(t)) \}$
2. $S_2 = \{ r(x, y), p_0(t) \lor \neg p_1(x), p_1(f(y)) \lor \neg r(y, f(z)), \neg p_0(x) \}$
3. $S_3 = \{ s(x, t, f(z)) \lor p(t), r(z, f(y)) \lor \neg p(z), \neg r(t, f(x)), \neg s(x, y, x) \}$