Exercise 1. Consider the following model $\mathcal{M} = (W, R, V)$:

![Diagram of model $\mathcal{M}$]

where all the propositional variables made true at a world are explicitly written in the circle related to the world. For example, we have that $\mathcal{M}, w_1 \models p$ and $\mathcal{M}, w_3 \models q$ while $\mathcal{W}, w_4 \not\models q$ and $\mathcal{M}, w_0 \not\models p$. Are the following statements true or false? In each case justify your answer.

1. $\mathcal{M}, w_0 \models \Box \Diamond q$
2. $\mathcal{M}, w_3 \models \Box \neg p$
3. $\mathcal{M}, w_2 \models \Box \Diamond \Diamond (p \land q)$
4. $\mathcal{M}, w_0 \models \Box (p \lor q)$
5. $\mathcal{M}, w_3 \models \Diamond (p \rightarrow q)$
6. $\mathcal{M} \models \Box p \rightarrow p$
7. $\mathcal{M} \models \Box (\Box p \rightarrow \Diamond (p \lor \neg \Box q))$
8. $(\mathcal{W}, R) \models \Box p \rightarrow \Diamond p$
9. $(\mathcal{W}, R) \models \Box p \rightarrow p$

Exercise 2. For each of the following formulae determine if it is valid. If it is justify your claim. If it is not provide a countermodel and then determine if it is satisfiable. If it is provide a model. If it is not justify your claim.

1. $\Diamond (p \land q) \rightarrow (\Diamond p \land \Diamond q)$
2. $\Box (p \lor q) \rightarrow (\Box p \lor \Box q)$
3. $\Diamond (p \land \neg p)$
4. $\Box (p \land q) \leftrightarrow (\Box p \land \Box q)$
5. $(\neg \Diamond p \land \neg \Box \neg p)$
6. $(\Box p \land \Box \neg p) \lor \Diamond \top$
Exercise 3. Let $\mathcal{M} = (W, R, V)$ be a Kripke model and $\phi$ a formula. Prove the following: if $\mathcal{M} \models \phi$, then for every $w \in W$ we have $\mathcal{M}, w \models \Box \neg \phi$ or $\mathcal{M}, w \models \Diamond \Box \phi$.

Exercise 4. Show the following:

1. $\Box p \rightarrow p \models \neg p \rightarrow \Diamond \neg p$
2. $\Box p \rightarrow p, \Diamond \Box p \rightarrow \Box p \models \neg p \rightarrow \Box \Diamond \neg p$
3. $\Box p \rightarrow p \models \Diamond \Box p \rightarrow \Diamond p$

Exercise 5. Let $\Gamma$ be a set of formulae. Show the following:

1. the canonical model $\mathcal{M}_\Gamma$ based on the logic $K4$ is transitive.
2. the canonical model $\mathcal{M}_\Gamma$ based on the logic $K2$ is weakly directed.
3. the canonical model $\mathcal{M}_\Gamma$ based on the logic $K + \Box (\phi \rightarrow \Diamond \phi)$ is one-step reflexive, i.e. satisfies $\forall w, v \in W_c (wRv \rightarrow vRv)$. 

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