Canberra Computer Science Enrichment: Logic 1

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1 Basics

Last week, we've talked about the logic of *propositions* where *formulae* were constructed from atomic statements like 'it is raining' or just p or q using *connectives* $\wedge, \vee, \rightarrow$ as well as the constants $\top$ (true) and $\bot$ (false).

This week, we want to use 'Logic for Fun' (developed by John Slaney at the ANU) to solve logic, and other puzzles. The idea is the following:

1. we write down the given statements as logical formulae

2. we put them into Logic for Fun as constraints

3. we ask Logic for Fun for a solution, then interpret the solution to solve the problem.

2 Warming Up

Register. Grab a laptop and find the Logic for Fun site (Google e.g. 'l4f anu'). You need to register (top right hand corner), and we promise to keep your data safe and secure.

Translate. We're using a very simple example. Suppose we know that we're given the following facts:

<blockquote>Luna is sad if it rains. It rains if Luna is sad. It is not raining.</blockquote>

We transcribe this into logic as follows:

$$\text{rain} \rightarrow \text{sad} \qquad \text{sad} \rightarrow \text{rain} \qquad \neg\text{rain}$$

Here 'sad' and 'rain' are our atomic propositions. Because we stipulate the truth of all three formulae, we need to conjoin them with a logical and ($\wedge$).

Solve. Click the 'Solver' button at the top of the Logic for Fun tool. We don't worry about the sorts and the vocabulary yet, and just put the conjunctions of the three formulae, i.e.

$$(\text{rain} \rightarrow \text{sad}) \wedge (\text{sad} \rightarrow \text{rain}) \wedge \neg\, \text{rain}.$$

into the box labelled 'Constraints'. Note the trailing full stop ('.') which indicates the end of the input. If you click 'solve' (do it!), you will be presented with the solution.

Interpret. We get one 'model', and the solver tells us that rain must be false, and sad must be false, too.

This means that there is only one way (one situation) in which the formula can be true, and this happens when $\text{rain} = \bot$ and $\text{sad} = \bot$. If you want, you can check that both (a) the formula is true with the above

values of the propositions, and (b) that the formula is not true for any other values, using last week's truth tables.

What happens when you additionally stipulate that sad must also be true?

Expand. In the above example, we had two implications: rain → sad and sad → rain. When we think about the truth values of implication (do it!), we realise that the only way in which *both* implications can be true at the *same* time is when the variables rain and sad have the *same* truth values. In Logic for Fun, we can write this as `rain = sad` and therefore

$$(\texttt{rain = sad}) \wedge \neg \ \texttt{rain}$$

represents the same situation as the formula above. Try it in L4F. Also, we can use `!=` to stipulate that two formulae have different truth values.

3   Knights and Knaves

This is one of a series of puzzles in Raymond Smullyan's book "What is the Name of This Book". We are on an island, where everyone is either a knight or a knave. Knights always tell the truth; knaves always lie.

You meet two people (say, Alice and Bob), and Alice says "At least one of us is a knave.". Who is a knight and who is a knave?

First determine the atomic propositions you need to use to model this in logic. Then think about a way of coding the knowledge above as a logical formula. Put it into the Logic for Fun solver (don't forget the full stop), and use the solver to determine a solution. From the truth values of the variables, you should be able to determine a solution to the puzzle. Of course, you should check whether the solution you have obtained is really a solution to the problem!

Variation. You meet two more inhabitants: Charlie and Deb. Charlie tells you that he, or Deb, or both are knights. Deb says "Charlie could say that I am a knave".

4   Checking Logical Consequence

So far, we have used the solver to find solutions to problems. That is, we have determined that – given the background knowledge in the problem – that a given variable necessarily has a certain truth value. This can be generalised to the following question:

Is the formula ψ true in *all* situations in which the formula ϕ is true?

In this case, ψ is a *logical consequence* of ϕ. For example, If $\phi = p \wedge q$ and $\psi = q$, then we know that $\psi = q$ is true whenever $\phi = p \wedge q$ is true. Intuitively, ϕ asserts 'more' than ϕ. Note that ϕ and ψ can be a lot more complex!

Given formulae ϕ and ψ, how can we use a tool like L4F to check whether ψ is a logical consequence of ϕ?

5   The Winds and the Windows

This is a problem adapted from Lewis Carroll, Symbolic Logic. We are given the following statements:

- There is always sunshine when the wind is in the East.

- When it is cold and foggy, my neighbor practices the flute.

- When my fire smokes, I set the door open.

- When it is cold and I feel rheumatic, I light my fire.

- When the wind is in the East and comes in gusts, my fire smokes.

- When I keep the door open, I am free from headache.

- Even when the sun is shining and it is not cold, I keep my window shut if it is foggy.

- When the wind does not come in gusts, and when I have a fire and keep the door shut, I do not feel rheumatic.

- Sunshine always brings on fog.

- When my neighbor practices the flute, I shut the door, even if I have no headache.

- When there is a fog and the wind is in the East, I feel rheumatic

One of the following statements is a logical consequence of the list of statements above:

1. I feel rheumatic whenever the wind comes in gusts.

2. When the wind is in the east, I keep my window shut.

Can you tell which using L4F?

6   Knights, Knaves and Normals

There are three people, Anna, Bella, and Charlie. One is a knight, who always tells the truth. One is a knave, who always lies. One is a normal person, who sometimes tells the truth and sometimes tells lies. They each make one statement:

A. I am normal.

B. A's statement is true.

C. I am not normal.

Can you find out who is a knight, a knave and a normal?

Variation: If knights are of higher rank than normals, which in turn are of higher rank than knaves, and you meet Alice and Bob, who say:

A. I am of lower rank than B.

B. That is not true!

What can you conclude from their statements?